2018

AP Calculus AB/ Calculus BC

Sample Student Reponses and Scoring Notes

DRAFT

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CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

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2018 SCORING GUIDELINES — Version 1.0

Question 1

(a) $\int_0^{300} r(t)\,dt = 270$

According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$.

2 : { 1 : integral; 1 : answer }

(b) $20 + \int_0^{300} (r(t) - 0.7)\,dt = 20 + \int_0^{300} r(t)\,dt - 0.7 \cdot 300 = 80$

According to the model, 80 people are in line at time $t = 300$.

2 : { 1 : considers rate out; 1 : answer }

(c) Based on part (b), the number of people in line at time $t = 300$ is 80.

The first time t that there are no people in line is $300 + \frac{80}{0.7} = 414.286$ (or 414.285) seconds.

1 : answer

(d) The total number of people in line at time t, $0 \le t \le 300$, is modeled by $20 + \int_0^t r(x)\,dx - 0.7t$.

$r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298,\ t_2 = 166.574719$

t	People in line for escalator
0	20
t_1	3.803
t_2	158.070
300	80

The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line.

4 : { 1 : considers $r(t) - 0.7 = 0$; 1 : identifies $t = 33.013$; 1 : answers; 1 : justification }

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GG

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$$\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt$$

$$= 270 \text{ people}$$

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$$\int_0^{300} r(t)\,dt + 20 - \int_0^{300} .7\,dt$$

$$290 - .7t \Big]_0^{300} \qquad 290 - 210$$

80 people

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Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$$\int_0^{300} r(t)\,dt + 20 - \int_0^t .7\,dx = 0$$

$$290 - .7x\Big|_0^t = 0$$

$$-.7t = -290$$

$$\boxed{t = 414.286}$$

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$$44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 - .7 = 0$$

$$t = 166.575,\ 33.013$$

$$\int_0^0 (r(t) - .7)\,dt + 20 = 20$$

$$\boxed{t = 33.013}$$

$$\int_0^{33.013} (r(t) - .7)\,dt + 20 = 20 - 16.19656 \approx \boxed{4 \text{ people}}$$

$$\int_0^{166.575} (r(t) - .7)\,dt + 20 = 20 + 138.070114 \approx 158$$

$$\int_0^{300} (r(t) - .7)\,dt + 20 = 80$$

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TTT

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$\int_0^{300} r(t)dt = \int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt = 270$

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$20 + \int_0^{300} r(t)dt - \int_0^{300} 0.7dt = 80$

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Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

∵ At $T = 300$, there is 80 people in line.

Let ~~$t =$~~ ~~dt~~ k be the time at which there are no people in line.

$80 - \int_{300}^{k} 0.7\,dt = 0$ $\quad k = 414.285$

∴ The first time t that there ~~are~~ are no people in line is when $t = 414.285$

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

∵ When the number of people in line reaches a local minimum,

$r(t) - 0.7 = 0$.

∴ When $r(t) - 0.7 = 0$, $t = 33.013298$ or 166.57472.

t	number of people
0	20
33.013298	3.803437
166.57472	158.07014
300	80

16.196563

138.07014

∴ At $t = 33.013$, the number of people in line is a minimum.

The number of people at $t = 33.013$ is 4.

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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

rate
ppl/sec

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

-0.7 ppl/sec

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

people enter $= \int_0^{300} r(t)\,dt = \int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt =$ **270 people**

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

At $t=300$, 270 people enter the line, adding onto the 20 who were already in line at $t=0$

∴ A Total of $270+20 = 290$ people were in line at some time t during interval $t \in [0, 300]$

290 total people in line $-$ (rate people exit line)(time)

$= 290 - 0.7(300) =$ **80 people** in line @ $t=300$

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Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

AKA when remaining people in line have left

P_r = remaining people in line

$P_r = 0 = 80 - 0.7(t-300)$

$80 = 0.7(t-300)$

$t = 414.286$ sec.

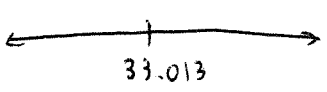
(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

# of ppl. in line min when rate of ppl. in line changing is 0

R = net rate of ppl. in line = rate enter − rate exit

$R(t) = 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 - 0.7 = 0$

$t = 33.013$



$t = 33.013$ is critical point on people function, and it's a relative minimum.

By Extreme Value Theorem, rel. crit pt. is absolute extrema, so it's the absolute minimum on the people function

People in line $= \int_0^{33.013} R(t)\,dt$

$= 6.913$

$\approx$ 7 people

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1. People enter a line for an escalator at a rate modeled by the function r given by

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where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$P = \int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt = 270$ people

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$P = P(0) + \int_0^{300} r(t)\,dt - \int_0^{300} 0.7\,dt$

$20 + 270 - 210 = 80$ people

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Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

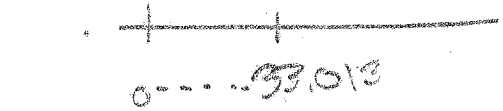
$P = 80 - \int_0^t 0.7\,dt = 0$ $\quad P = 80 - 0.7t = 0$

$80 = 0.7[t - 0]$

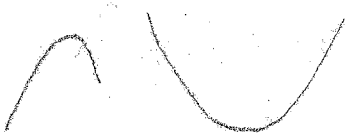
$t = \frac{80}{.7} = 114.286$ seconds

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this ~~time. Justify~~ your answer.

$P'(t) = r(t) - 0.7$



At $t = 33.013$ $r(t) - 0.7$ changes from negative to positive and $r'(t) > 0$ so the number of people is at a minimum



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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt$

$=$ 270 people enter the line.

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$20 + \int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 dt - \int_0^{300} 0.7 \, dt$

$= 20 + 270 - 210$

$= 80$

80 people are in line at $t = 300$.

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Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$20 + 270 = 0.7t$

$290 = 0.7t$

$t = 414.286$

∴ The first time is at 414.286 seconds

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$$r'(t) = 132\left(\frac{t}{100}\right)^2 \cdot \left(\frac{1}{100}\right)\left(1-\frac{t}{300}\right)^7 + 308\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^6 \cdot \frac{-1}{300}$$

$$= 1.32\left(\frac{t}{100}\right)^2\left(1-\frac{t}{300}\right)^7 - \frac{308}{300}\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^6$$

$$0 = \left(\frac{t}{100}\right)^2\left(1-\frac{t}{300}\right)^6\left[1.32\left(1-\frac{t}{300}\right) - \frac{308}{300}\left(\frac{t}{100}\right)\right]$$

$$0 = 1.32 - \frac{1.32t}{300} - \frac{308t}{30000}$$

$$\frac{1.32t}{300} + \frac{308t}{30\,000} = 1.32$$

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1. People enter a line for an escalator at a rate modeled by the function r given by

rate that people enter

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

people/sec

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$\int_0^{300} r(t)\,dt = 270$ people enter the line on the time interval $0 \le t \le 300$.

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$\left[20 + \int_0^{300} r(t)\,dt\right] - .7(300) = 80$ people are on the line at $t = 300$.

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Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$$\left[20 + \int_0^{300} r(t)\,dt\right] - .7(t) = 0$$

$$\left[20 + \int_0^{300} r(t)\,dt\right] = .7(t)$$

$$270 = .7(t)$$

$$t = 385.714 \text{ seconds}$$

The first time there are no people on line is at $t = 385.714$ seconds

---

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$$r'(t) = \left[44\left(\frac{t}{100}\right)^3 \cdot -\frac{1}{300}\left(1 - \frac{t}{300}\right)^6\right] + \left[\left(1 - \frac{t}{300}\right)^7 \cdot \frac{132}{100}\left(\frac{t}{100}\right)^2\right]$$

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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$R = \int_0^{300} \left(44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7\right) dt$ +20 into graphing calculator

$= 270 + 20$

$= 290$

∴ 290 people enter the line for the escalator during the time interval $0 \le t \le 300$.

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

From a, 290 ppl enter line.

300 s, −0.7 ppl /sec.

$$\begin{array}{r} 300 \\ \times\ 0.7 \\ \hline 210.0 \end{array}$$

$290 - 210.0 = 80$

∴ 80 people are in line for the escalator at $t = 300$

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Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$t > 300$

From b, 80 ppl in line at $t = 300$ s.

ppl leave at 0.7 ppl/sec.

$\frac{80 \text{ ppl}}{0.7 \text{ ppl/sec}} \approx 114$ sec

$\therefore$ At $t = 414$, there are no people in line for the escalator.

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$$44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 = 0$$

$$\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 = 0$$

$\frac{t}{100} = 0$

$t = 0$

when $t = 0$, there are 20 ppl.

$1 - \frac{t}{300} = 0$

$300 - t = 0$

$t = 300$

when $t = 300$, there's 80 ppl

$\therefore$ At $t = 0$, the number of people in line will be a minium, which is 20.

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EEE

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$\int_0^{300} r(t)\,dt =$ 270 people

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(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$1 - .7 = .3$ remaining each second

$\left(20 + \int_0^{300} r(t)\,dt\right)(.3)$

$(20 + 270)(.3) =$ 87 people!

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Continue question 1 on page 5.

EEE

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

$.7t = 87$ | 124.2857 seconds

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$r(t)$ = a minimum at | $t = 235.1064$ seconds (

$r(235.1064)$ = | 28 people

This is the point when the rate of people exiting meets up to the initial condition, so it is now only amount of people coming in vs. going.

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1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

$\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt + 20 - \int_0^{300} 0.7\, dt$

$= 290$ people $- 210$

$= 80$ people

$0.7t\Big|_0^{300}$

0.7×300

$= 210 - 0$

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(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

$\int_{300}^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt + 20$

$= 0 + 20$

$= 20$ people.

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Continue question 1 on page 5.

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

Exit $= r(t) = 0.7$

$\int r(t) = \int 0.7 = 0.7t$

$290 - 0.7t = 0$

$t = 414.3$

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$r(t) = 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 = 0$

$t = 0$, number of people in line a minimum.

$\int_0^0 r(t) + 20$

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Question 1

Sample Identifier: GG
Score: 9

- **The response earned 2-2-1-4.**
- In part (a) the response earned the first point for the integral $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt$. The response earned the second point for the answer 270.
- In part (b) the response earned the first point with the term $-\int_0^{300}.7dt$. The response earned the second point for the answer 80.
- In part (c) the response earned the point for the answer 414.286.
- In part (d) the response earned the first point in the first line for the equation $44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7-.7=0$. The response earned the second point in the second line for identifying 33.013. The response earned the third point for the answers 33.013 and 4. Units were not required to earn this point or any other point in this problem. The response earned the fourth point for the successful candidates test supported by the presentation of the definite integrals. NOTE: The time and people values, presented in the candidates test, are correctly rounded from the scoring guidelines.

Sample Identifier: TTT
Score: 8

- **The response earned 2-2-1-3.**
- In part (a) the response earned the first point for the integral $\int_0^{300} r(t)dt$. The response earned the second point for the answer 270.
- In part (b) the response earned the first point with the term $-\int_0^{300} 0.7dt$. The response earned the second point for the answer 80.
- In part (c) the response earned the point for the answer 414.285.
- In part (d) the response earned the first point for the equation $r(t)-0.7=0$ in the second line. The response earned the second point with 33.013298 in the third line. The response earned the third point for the answers in the last line. The response did not earn the fourth point because though the response has a candidates test, the response lacks a definite integral to show how 3.803437 and 158.07014 were calculated.

Sample Identifier: ZZ
Score: 7

- **The response earned 2-2-1-2.**
- In part (a) the response earned the first point for the integral $\int_0^{300} r(t)dt$. The response earned the second point for the answer 270.
- In part (b) the response earned the first point for considers rate out on the sixth line with the term $-0.7(300)$. The response earned the second point for the answer 80.
- In part (c) the response earned the point for 414.286.
- In part (d) the response earned the first point for the equation $44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7-0.7=0$ in the third line. The response earned the second point with 33.013 in the fourth line. The response does not earn the third point for answers since the number of people, 7, is incorrect. The response is not eligible to earn the fourth point since an incorrect answer cannot be justified.

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Question 1 (continued)

Sample Identifier: WWW
Score: 6

- **The response earned 2-2-0-2.**
- In part (a) the response earned the first point for the integral $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt$. The response earned the second point for the answer 270.
- In part (b) the response earned the first point with the term $-\int_0^{300} 0.7dt$. The response earned the second point for 80. The expression 20 + 270 – 210 would have earned the second point with no simplification. The student chose to simplify, did so correctly, and thus earned the second point.
- In part (c) the response did not earn the point. The answer 114.286 is incorrect because 300 was not added.
- In part (d) the response earned the first point for considers $r(t) - 0.7 = 0$ by writing, "$r(t) - 0.7$ changes from negative to positive" on the second and third lines. The response earned the second point with 33.013 on the second line. The response does not earn the third point since the answer 4 is missing. The response is not eligible to earn the fourth point because an incomplete answer cannot be justified.

Sample Identifier: S
Score: 5

- **The response earned 2-2-1-0.**
- In part (a) the response earned the first point for the integral $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt$. The response earned the second point for the answer 270.
- In part (b) the response earned the first point with the term$-\int_0^{300} 0.7dt$. The response earned the second point for 80. The expression 20 + 270 – 210 would have earned the second point with no simplification. The student chose to simplify, did so correctly, and thus earned the second point.
- In part (c) the response earned the point for the answer 414.286.
- In part (d) the response did not earn any points. The response presents a correct derivative for $r(t)$, but $r'(t)$ is not relevant to the question.

Sample Identifier: T
Score: 4

- **The response earned 2-2-0-0.**
- In part (a) the response earned the first point for the integral $\int_0^{300} r(t)dt$. The response earned the second point for the answer 270.
- In part (b) the response earned the first point for considers rate out with the term $-.7(300)$. The response earned the second point for the answer 80.
- In part (c) the response did not earn the point because 385.714 is incorrect.
- In part (d) the response did not earn any points. The response presents a correct derivative for $r(t)$, but $r'(t)$ is not relevant to the question.

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Question 1 (continued)

Sample Identifier: V
Score: 3

- **The response earned 1-2-0-0.**
- In part (a) the response earned the first point for $\int_0^{300}\left(44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7\right)dt$. The response does not earn the second point since 290 is the incorrect answer.
- In part (b) the response earned the first point with the term – 210.0 in the third line. The response earns the second point for 80. The expression 290 – 210 would have earned the second point with no simplification. The student chose to simplify, did so correctly, and thus earned the second point.
- In part (c) the response does not earn the point because 414 is not presented to three decimal places.
- In part (d) the response did not earn any points because it only considers $r(t) = 0$ rather than $r(t) - 0.7 = 0$.

Sample Identifier: EEE
Score: 2

- **The response earned 2-0-0-0.**
- In part (a) the response earned the first point for $\int_0^{300} r(t)dt$. The response earned the second point for the answer 270.
- In part (b) the response did not earn any points. The rate at which people exit the line is mishandled.
- In part (c) the response did not earn the point. The value 124.2857 is neither correct nor is it consistent from the response answer in part (b). Since the response answer in part (b) was 87, the number of seconds would have been 424.2857.
- In part (d) the response does not consider $r(t) - 0.7 = 0$ and did not earn any points.

Sample Identifier: E
Score: 1

- **The response earned 1-0-0-0.**
- In part (a) the response earned the first point for $\int_0^{300} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 dt$. The response did not earn the second point since 80 is not the correct answer.
- In part (b) the response did not earn the first point. In this part, the response was not eligible to earn the second point without earning the first point. The complete solution for part (b) is presented in part (a), but there is no appeal to that solution in part (b).
- In part (c) the response does not earn the point because 414.3 is not presented to three decimal places.
- In part (d) the response did not earn any points since the student considers $r(t) = 0$ instead of $r(t) - 0.7 = 0$.

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

END OF PART A OF SECTION II

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2018 SCORING GUIDELINES — Version 1.0

Question 2

(a) $v'(3) = -2.118$

The acceleration of the particle at time $t = 3$ is -2.118.

1 : answer

(b) $x(3) = x(0) + \int_0^3 v(t)\,dt = -5 + \int_0^3 v(t)\,dt = -1.760213$

The position of the particle at time $t = 3$ is -1.760.

3 : { 1 : $\int_0^3 v(t)\,dt$; 1 : uses initial condition; 1 : answer }

(c) $\int_0^{3.5} v(t)\,dt = 2.844$ (or 2.843)

$\int_0^{3.5} |v(t)|\,dt = 3.737$

The integral $\int_0^{3.5} v(t)\,dt$ is the displacement of the particle over the time interval $0 \le t \le 3.5$.

The integral $\int_0^{3.5} |v(t)|\,dt$ is the total distance traveled by the particle over the time interval $0 \le t \le 3.5$.

3 : { 1 : answers; 2 : interpretations of $\int_0^{3.5} v(t)\,dt$ and $\int_0^{3.5} |v(t)|\,dt$ }

(d) $v(t) = x_2'(t)$

$v(t) = 2t - 1 \Rightarrow t = 1.57054$

The two particles are moving with the same velocity at time $t = 1.571$ (or 1.570).

2 : { 1 : sets $v(t) = x_2'(t)$; 1 : answer }

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

acceleration $= v'(t)$

$v'(3) = \boxed{-2.118}$

(b) Find the position of the particle at time $t = 3$.

position $= -5 + \displaystyle\int_0^3 v(t)\,dt = \boxed{-1.760}$

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Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = 2.844$ ⇒ total displacement of particle from time=0 to time=3.5

$\int_0^{3.5} |v(t)|\,dt = 3.737$ ⇒ total distance traveled from time=0 to time=3.5

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$x_2'(t)$ = velocity of particle 2

$x_2'(t) = 2t - 1$

$2t - 1 = v(t)$

$\boxed{t = 1.571}$

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$v'(t) = a(t)$

$a(3) = \frac{d}{dx}\left(\frac{10\sin(.4t^2)}{t^2 - t + 3}\right) = \boxed{-2.118}$

(b) Find the position of the particle at time $t = 3$.

$p(t) = \int v(t)$

$p(3) = \int_0^3 v(t)\,dt = 3.240$ units to the right

$-5 + 3.24 = -1.76$

the particle is at position $x = -1.76$ at $t = 3$

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Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = \boxed{2.844}$ → The net distance the particle moves from its starting position

$\int_0^{3.5} |v(t)|\,dt = \boxed{3.737}$ → the total distance the particle moves from its starting position

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$x_2(t) = t^2 - t$

$v_2(t) = 2t - 1$

$2t - 1 = \dfrac{10\sin(.4t^2)}{t^2 - t + 3}$

Intersect at $\boxed{t = 1.571}$

When $t = 1.571$, the particles are moving with the same velocity

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$a(3) = -2.118$

(b) Find the position of the particle at time $t = 3$.

$$-5 + \int_0^3 \frac{10\sin(0.4t^2)}{t^2 - t + 3}\,dt = -1.760$$

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Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = \boxed{2.844}$

This is the change in position of the particle from $0 \le t \le 3.5$

$\int_0^{3.5} |v(t)|\,dt = \boxed{3.737}$

This is the total distance that the particle traveled from $0 \le t \le 3.5$.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$x_2(t) = t^2 - t$

$v_2(t) = 2t - 1$

$v_2(t) = v_1(t)$ at $t = 1.57054$ because that is where the two velocity graphs intersect.

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$a(t) = v'(t)$

$a(3) = -2.118$

(b) Find the position of the particle at time $t = 3$.

$P(t) = -5 + \int_0^3 v(t)\,dt$

$P(3) = -1.760$

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(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = 2.844$

$\int_0^{3.5} |v(t)|\,dt = 3.737$

$\int_0^{3.5} v(t)\,dt$ is the displacement of the particle.

$\int_0^{3.5} |v(t)|\,dt$ is the net distance the particle traveled.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$v_2(t) = 2t - 1$

$v(t) = v_2(t)$ at $t = 2.122$

The two particles are moving at the same velocity at $t = 2.122$.

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$v'(t) = a(t)$

$a(3) = 0.409$

(b) Find the position of the particle at time $t = 3$.

$$x(0) + \int_0^3 v(t)\,dt = x(3)$$

$-5 + 6.382$

$x(3) = 1.382$

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Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = 8.372$

The total displacement of the particle on the time interval $0 \le t \le 3.5$

$\int_0^{3.5} |v(t)|\,dt = 8.372$

The total distance traveled by the particle on the time interval $0 \le t \le 3.5$

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$x_2'(t) = v(t)$

$x_2'(t) = 2t - 1$

$2t - 1 = \dfrac{10\sin(0.4t^2)}{t^2 - t + 3}$

The two particles intersect at $t = 2.165$

The particles are moving with the same velocity only at $t = 2.165$

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$$v(t) = \frac{10\sin(0.4t^2)}{t^2 - t + 3}$$

$$v'(3) = -2.118$$

(b) Find the position of the particle at time $t = 3$.

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(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} \left(\frac{10\sin(0.4t^2)}{t^2-t+3}\right) dt = 2.5439$

$\int_0^{3.5} |v(t)|\,dt = 3.7371$

$\int_0^{3.5} v(t)\,dt$ is the distance the particle travels from the origin

$\int_0^{3.5} |v(t)|\,dt$ is the total distance travelled by the particle

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$x_2(t) = t^2 - t$

$x_2'(t) = 2t - 1$

$x_2'(t) = v(t)$

$t = 1.5705$

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

Acceleration = derivative of velocity

Derivative:

$$\frac{dy}{dx}\left(\frac{10\sin(0.4t^2)}{t^2 - t + 3}\right)\Bigg|_{t=3} = -3.064$$

$-3.064 + (-5) = \boxed{-8.064}$

(b) Find the position of the particle at time $t = 3$.

Position = displacement = integral

$$\int_0^3 \frac{10\sin(0.4t^2)}{t^2 - t + 3}\,dt \approx 12.83$$

$12.83 + (-5) = \boxed{7.83}$

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Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)\,dt = 12.32 + (-5) = \boxed{7.32}$

$\int_0^{3.5} |v(t)|\,dt = 12.32 + (-5) = \boxed{7.32}$

In the context of the problem $\int_0^{3.5} v(t)\,dt$ represents the ~~distance traveled~~ displacement and $\int_0^{3.5} |v(t)|\,dt$ represents distance traveled.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$x_2(t) = t^2 - t = v(t) = \dfrac{10\sin(.4t^2)}{t^2 - t + 3}$

≈ 2.2

$\boxed{t \approx 2.2}$

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$$a = \frac{(t^2-t+3)(10\cos(0.8t)) - 10\sin(0.4t^2)t-1}{(t^2-t+3)^2}$$

$$a = -0.668$$

(b) Find the position of the particle at time $t = 3$.

$$\int \frac{10\sin(0.4t^2)}{t^2-t+3}\,dt$$

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Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$$\int_0^{3.5} \frac{10\sin(0.4t^2)}{t^2-t+3}\,dt = 2.844$$

$$\int_0^{3.5} \left|\frac{10\sin(0.4t^2)}{t^2-t+3}\right|\,dt = 3.737$$

Both integrals are increasing with respect to the function. The integral $\int_0^{3.5} |v(t)|\,dt$ is increasing at a greater rate than $\int_0^{3.5} v(t)\,dt$.

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

$$v_2 = 2t - 1$$

$$2t - 1 = \frac{10\sin(0.4t^2)}{t^2-t+3}$$

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2. A particle moves along the x-axis with velocity given by $v(t) = \dfrac{10\sin\left(0.4t^2\right)}{t^2 - t + 3}$ for time $0 \le t \le 3.5$.

The particle is at position $x = -5$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 3$.

$$v(t) = \frac{10\sin(0.4t^2)}{t^2 - t + 3}$$

$$a(t) = \frac{(t^2 - t + 3)\cdot 10\cos(0.4t^2)\cdot 0.8t - (10\sin(0.4t^2))(2t - 1)}{(t^2 - t + 3)^2}$$

$a(3) = -2.118$

(b) Find the position of the particle at time $t = 3$.

$$v(t) = \int \frac{10\sin(0.4t^2)}{t^2 - t + 3} \qquad \int v(t) = p(t)$$

$p(t) = 3.24$

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Continue question 2 on page 7.

(c) Evaluate $\int_0^{3.5} v(t)\,dt$, and evaluate $\int_0^{3.5} |v(t)|\,dt$. Interpret the meaning of each integral in the context of the problem.

$\int_0^{3.5} v(t)dt$ is which way the particle is going and $\int_0^{3.5} |v(t)|\,dt$ is how fast the velocity ~~is in~~ ~~the~~ of the particle is

(d) A second particle moves along the x-axis with position given by $x_2(t) = t^2 - t$ for $0 \le t \le 3.5$. At what time t are the two particles moving with the same velocity?

they are both moving the same velocity at $t = 0$ and at $t = 1.999$

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AP® SAMPLE STUDENT RESPONSES AND SCORING NOTES
2018 AP CALCULUS AB

Question 2

Sample Identifier: A
Score: 9

- **The response earned 1-3-3-2.**
- In part (a) the point was earned with the equation $v'(3) = -2.118$.
- In part (b) the first point was earned with the presentation of $\int_0^3 v(t)dt$. The second point was earned through the addition of -5 to that integral. The third point was earned with the correct answer of -1.760.
- In part (c) the values of the two integrals on the left side are correct and earned the first point. Both of the two interpretation points were earned as the two integrals are interpreted correctly as (total) displacement and total distance traveled and both references to the time interval are considered correct.
- In part (d) the first point was earned with the presentation of both of the two equations $x'_2(t) = 2t - 1$ and $2t - 1 = v(t)$. The answer point was earned with the last line.

Sample Identifier: B
Score: 8

- **The response earned 1-3-2-2.**
- In part (a) the point was earned because there is an indication that $v'(t)$ is being used to find the correct answer -2.118. The poor communication using $\frac{d}{dx}$ instead of $\frac{d}{dt}$ concerning $v'(t)$ in the second line was not penalized.
- In part (b) the first point was earned with the presentation of $\int_0^3 v(t)dt$ in the second line. The second point was earned in the third line through the addition of -5 to the value of that integral. The third point was earned with the correct answer of -1.76. Note that -1.76 and -1.760 are considered equal, so -1.76 does not result in a decimal presentation error.
- In part (c) the values of the two integrals on the left side are correct and earned the first point. Only one of the two interpretation points was earned. The integrals are correctly identified as "net distance" (interpreted as displacement) and total distance traveled, but the time interval was not referenced.
- In part (d) the first point was earned with the third equation. The correct answer earned the second point.

Sample Identifier: CC
Score: 7

- **The response earned 0-3-3-1.**
- In part (a) the point was not earned because there is no stated connection between -2.118 and $v'(3)$. Identifying an answer as $a(3)$ was insufficient without an explicit statement that $a(t) = v'(t)$.
- In part (b) the first point was earned with $\int_0^3 \frac{10\sin(0.4t^2)}{t^2-t+3}dt$. The second point was earned through the addition of -5 to that integral. The third point was earned with the correct answer of -1.760.
- In part (c) the values of the two integrals on the left side are correct and earned the first point. Both of the two interpretation points were earned as the two integrals are interpreted correctly as "change in position", another way to describe displacement, and total distance traveled, and both have a reference to the time interval $0 \le t \le 3.5$.
- In part (d) the first point was not earned because there is no declaration that $v_1(t) = v(t)$. If the response had indicated that $v_1(t)$ was the velocity of the first particle in the stem of the problem,

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Question 2 (continued)

the first point would have been earned. The second point was earned with the equation $t = 1.57054$.

Sample Identifier: D
Score: 6

- **The response earned 1-3-1-1.**
- In part (a) the point was earned by the correct answer of -2.118 and the connection of it to $v'(t)$ found in the first equation.
- In part (b) the first point was earned with the presentation of $\int_0^3 v(t)dt$. The second point was earned through the addition of -5 to that integral. The third point was earned with the correct answer of -1.760. In the first line, the use of $p(t)$ rather than $p(3)$ was not penalized.
- In part (c) the integral values are given correctly and earned the first point. The first interpretation identifies the first integral correctly as displacement but the interval of time $0 \le t \le 3.5$ is not referenced. The interpretation of the second integral as "net displacement" is incorrect and also does not reference the time interval. The combination of these errors meant that the last two points were not earned.
- In part (d) the two equations $v_2(t) = 2t - 1$ and $v(t) = v_2(t)$ earned the first point. The answer of $t = 2.122$ is incorrect and did not earn the answer point.

Sample Identifier: E
Score: 5

- **The response earned 0-2-2-1.**
- In part (a) the point was not earned because the answer of 0.409 is incorrect.
- In part (b) the first point was earned with the presentation of the $\int_0^3 v(t)dt$ in the first equation. The second point was earned on the second line with the replacement of $x(0)$ in the first equation with the initial value of -5. The answer is not correct and did not earn the third point.
- In part (c) the first point was not earned because the values for both integrals are incorrect. The interpretations given are nearly identical to the scoring guideline interpretations and earned both of the last two points.
- In part (d) the first point was earned with the first equation $x'_2(t) = v(t)$. The answer of $t = 2.165$ is incorrect and did not earn the second point.

Sample Identifier: F
Score: 4

- **The response earned 1-0-1-2.**
- In part (a) the last line earned the point.
- In part (b) there is no response, so no points were earned.
- In part (c) the first point was earned because the values of the two integrals are correct. The last two points were not awarded because there is no reference to the time interval and the first integral is incorrectly interpreted as the "distance the particle travels from the origin."
- In part (d) the third equation earned the first point. The correct answer in the last equation earned the second point.

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Question 2 (continued)

Sample Identifier: C
Score: 3

- **The response earned 0-2-1-0.**
- In part (a) the point was not earned because the answer of -8.064 is incorrect.
- In part (b) the first point was earned with the integral presented. The second point was earned with the addition of -5 to value that was computed for the integral. The third point was not earned because the answer is incorrect as a result of the incorrect computation of the value of the integral.
- In part (c) the first point was not earned because the values of the two integrals are not correct. While the two integrals are interpreted correctly as displacement and distance traveled, only one of the last two points was earned because of no reference to the interval of time.
- In part (d) the first point was not earned because the response never equates $x'_2(t)$ and $v(t)$. The second point was not earned because the answer of 2.2 is not correct.

Sample Identifier: G
Score: 2

- **The response earned 0-0-1-1.**
- In part (a) the point was not earned because the answer -0.668 is incorrect.
- In part (b) the first point was not earned because the integral does not have the limits of integration. There is no further work, so the last two points were not earned.
- In part (c) the first point was earned because of the correct values for the two integrals. There is no interpretation that refers to displacement or distance traveled, and no time interval is referenced so the last two points were not earned.
- In part (d) the point was earned for setting the derivative, $2t - 1$, of $x_2(t)$ equal to the expression for $v(t)$. The second point was not earned because there is no answer.

Sample Identifier: H
Score: 1

- **The response earned 1-0-0-0.**
- In part (a) the point was earned. While there is no explicit reference to $v'(t)$, the correct expression for $v'(t)$ is given in the second equation and identifies it as $a(t)$. The third equation gives the correct answer.
- In part (b) the first point was not earned because the integral given is an indefinite integral. There is no reference to the initial condition and the answer of 3.24 is incorrect, so neither of the last two points were earned.
- In part (c) the first point was not earned because the integral values are not given. The interpretations for both integrals are incorrect and there is no reference to the time interval, so the last two points were not earned.
- In part (d) no points were earned because $v(t)$ is not set equal to $x'_2(t)$ and there is no correct answer.

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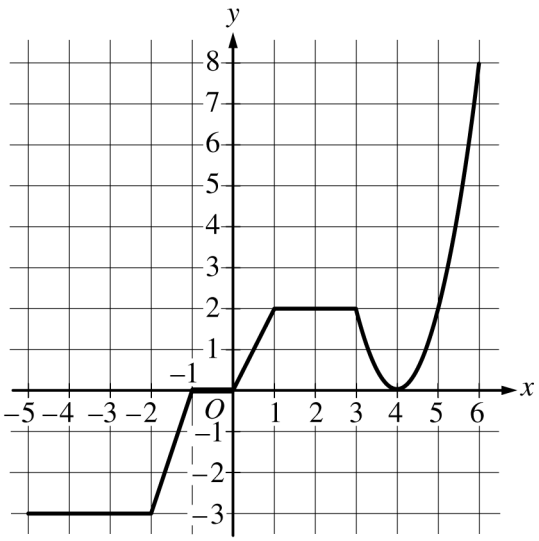
CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

   (a) If $f(1) = 3$, what is the value of $f(-5)$?

   (b) Evaluate $\int_1^6 g(x)\,dx$.

   (c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

   (d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

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Question 3

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^1 g(x)\,dx$
$= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$
$= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$
$= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$

$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of f has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

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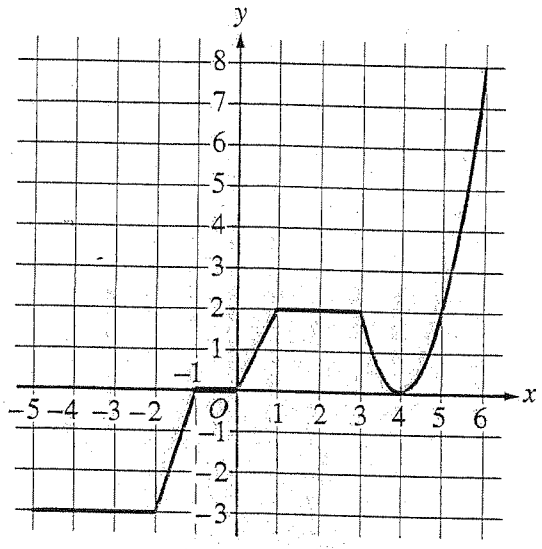

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$$f(-5) = 3 - \frac{1}{2}(1)(2) + \frac{1}{2}(3)(1) + 3(3)$$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$$\int_1^6 g(x)\,dx = 4 + \int_3^6 2(x-4)^2\,dx$$

$$= 4 + \frac{2(x-4)^3}{3}\Big|_3^6 = 4 + \frac{2(2)^3}{3} + \frac{2}{3}$$

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Continue question 3 on page 15.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

$(0,1)$ $(4,6)$ because f' or g is > 0

and because f'' or g' is > 0

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

$x = 4$, f'' or g' changes from negative to positive

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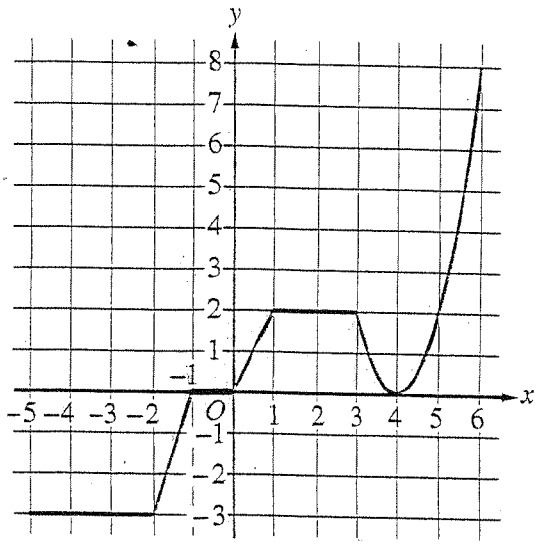

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$f(1)=3$

$f(-5) = f(1) + \int_1^{-5} g(x)\,dx = 3 - \int_{-5}^{1} g(x)\,dx = 3 - \left((-9) - \frac{3}{2} + 1\right)$

$= 3 - \left(-\frac{16}{2} - \frac{3}{2}\right)$

$= 3 + \frac{19}{2} = \frac{6}{2} + \frac{19}{2} = \frac{25}{2}$

$f(-5) = \frac{25}{2}$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$\int_1^6 g(x)\,dx = \int_1^3 2\,dx + \int_3^6 \left(2(x-4)^2\right) dx = 4 + 2\int_3^6 (x^2 - 8x + 16)\,dx$

$= 4 + 2\left(\frac{1}{3}x^3 - 4x^2 + 16x\right)\Big|_3^6$

$= 4 + 2\left((72 - 144 + 96) - \left(\frac{27}{3} - 36 + 48\right)\right)$

$= 4 + 2\left(24 - \left(\frac{27}{3} + \frac{36}{3}\right)\right) = 4 + 2(24 - 21) = 4 + 6 = 10$

$\int_1^6 g(x)\,dx = 10$

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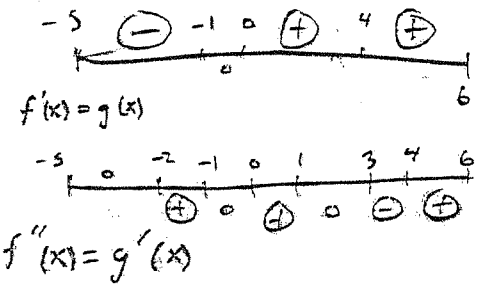
Continue question 3 on page 15.

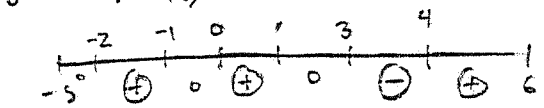
(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

increasing and concave up!
$(0,1) \cup (4,6)$



(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

Point of inflection where
$x = 4$



There is a point of inflection at $x = 4$ because that is the point where $f''(x) = 0$ or does not exist and also where there is a change in sign of $f''(x)$.

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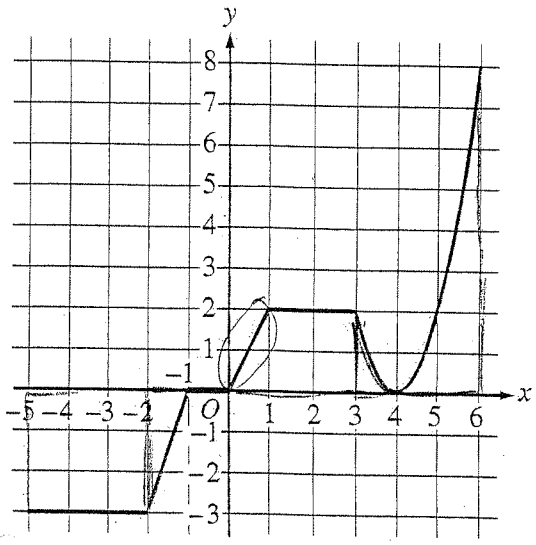
C



Graph of $g = f'$

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$$f(-5) = f(1) - \int_{-5}^{1} f'(x)\,dx$$

$$= 3 - \int_{-5}^{1} g(x)\,dx$$

$$= 3 - \left(\tfrac{1}{2}(1)(-3) + (-3)(-3)\right)$$

$$= 3 - \left(-\tfrac{3}{2} - 6\right)$$

$$= 3 - \left(-\tfrac{3}{2} - \tfrac{12}{2}\right)$$

$$= 3 - -\tfrac{15}{2}$$

$$= 3 + \tfrac{15}{2}$$

$$= \tfrac{6}{2} + \tfrac{15}{2}$$

$$= \boxed{\tfrac{21}{2}}$$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$$\int_1^6 g(x)\,dx =$$

$$= \int_1^3 g(x)\,dx + \int_3^6 2(x-4)^2\,dx$$

$$= 6 + 2\left(\tfrac{1}{3}\right)(x-4)^3 \Big|_3^6$$

$$= 6 + \left[\tfrac{2}{3}(2)^3 - \tfrac{2}{3}(-1)^3\right]$$

$$= 6 + \left[\tfrac{2}{3}(8) + \tfrac{2}{3}\right]$$

$$= 6 + \tfrac{16}{3} + \tfrac{2}{3}$$

$$= 6 + \tfrac{18}{3} = \boxed{12}$$

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Continue question 3 on page 15.

C

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

f is increasing when $f' = g$ is > 0

f is concave up when $f' = g$ is increasing

f is both increasing and concave up on $(0,1) \cup (4,6)$ b/c on these intervals $f' = g$ is positive and increasing

---

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

f has point of inflection at $x=4$ b/c $f' = g$ changes from decreasing to increasing at $x = 4$

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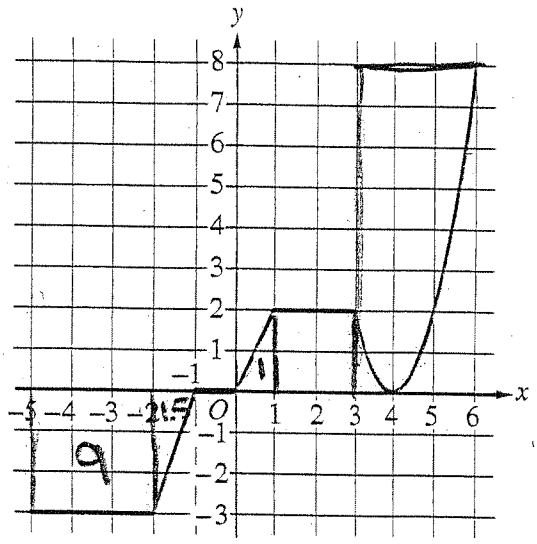

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$. $g(x)=F'(x)$

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$F = 3 + \int_1^{-5} g(x)\,dx$

$F = 3 - 1 + 1.5 + 9 = 12.5$

(b) Evaluate $\int_1^6 g(x)\,dx$. say that $x=6$

$\int_1^6 g(x)\,dx = 4 + [24 - (2(x-4)^2)]$

$= 4 + [24 - 8] = 20$

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Continue question 3 on page 15.

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D

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

F' = graph of g value

F" = graph of g slope

X = (0,1) & (4,6)

At these intervals, the value & slope of g are both positive, which are also F' & F"

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

F" = graph of g slope = 0

x = 4

The slope of g equals zero, switching from negative to positive, which is also F"

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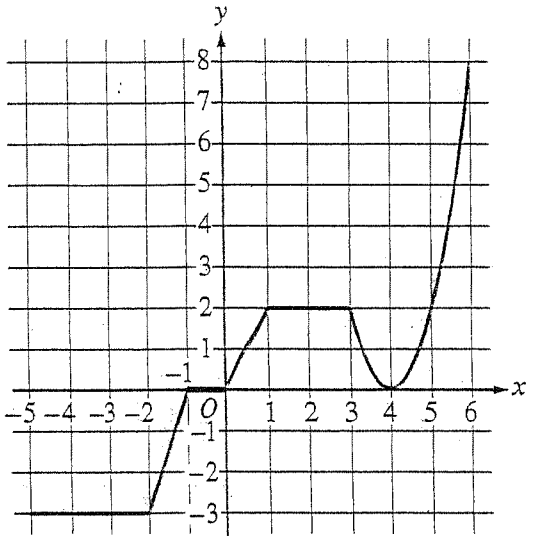

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$f(-5) = f(1) + \int_1^{-5} f'(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$

$= (3) - \left(\frac{(-3)(3)}{2} - \frac{(3)(1)}{2} + \frac{(2)(1)}{2}\right) = \boxed{-6.5}$

$-(-9 - 3/2 + 1)$

$-(-9.5) = 9.5$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$\int_1^6 g(x)\,dx = \frac{(2)(2)}{2} + \int_3^6 2(x-4)^2\,dx = 2 + 6 = \boxed{8}$

$+ \frac{2}{3}(x-4)^3 \Big]_3^6$

$+\left(\left(\frac{2}{3}(2)^3\right) - \left(\frac{2}{3}(-1)^3\right)\right)$

$16/3 + 2/3 = 6$

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Continue question 3 on page 15.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

f increasing: $f' > 0$, $g > 0$

f concave up: $f'' > 0$, $g' > 0$

$g > 0$ for $(0,4) \cup (4,6)$

$g' > 0$ for $(-2,-1) \cup (0,1) \cup (4,6)$

$(g > 0) \& (g' > 0)$ for $\boxed{(0,1) \cup (4,6)}$

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

Point of inflection: f' goes from positive to negative or from negative to positive, concavity changes.

$\boxed{x = 4}$ The slope, g, from the left is negative and from the right is positive. No other point has this property.

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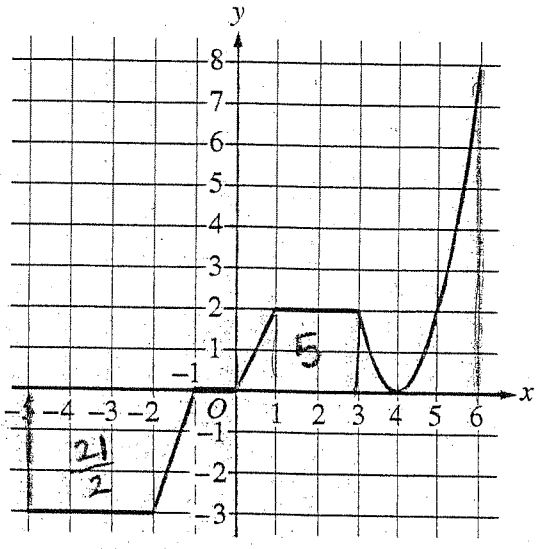
F



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$f(-5) = 3$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$4 + \int_3^6 2(x-4)^2\,dx$

$4 + \left[\frac{2}{3}(x-4)^3\right]_3^6 = 4 + \left(\frac{16}{3} + \frac{2}{3}\right) = 13$

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Continue question 3 on page 15.

F

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(0,1) and (4,6) because the graph of f is both positive and increasing

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

x=4 because the graph changes sign

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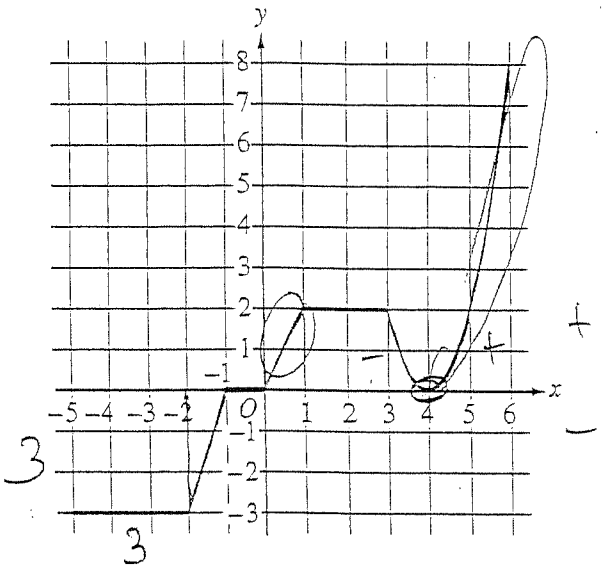

Graph of g

$g(x) = f'(x)$

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$f(-5) = f(1) + f(-5)$

$3 + \left[3^2 + \frac{1}{2}(1)(-3)\right]$

$3 + 9 - \frac{3}{2}$

$3 + \frac{18}{2} - \frac{3}{2}$

$3 + \frac{15}{2} = \frac{6}{2} + \frac{15}{2} = \frac{21}{2}$

$f(-5) = \frac{21}{2}$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$\int_1^6 2(x-4)^2\,dx$

$\frac{2}{3}(x-4)^3 \Big|_1^6$

$\frac{2}{3}(6-4)^3 - \frac{2}{3}(1-4)^3$

$\frac{16}{3} + \frac{54}{3} = 70$

$\int_1^6 g(x)\,dx = \frac{70}{3}$

$\frac{54}{3} + \frac{16}{3} = \frac{70}{3}$

$27 \times 2 = 54$

$54 + 16 = 70$

70

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Continue question 3 on page 15.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

$g(x) = f'(x)$

f is increasing and concave up on (0, 1) and (4, 6) because $f'(x) > 0$ and $f''(x) > 0$.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

There is a point of inflection at $x = 4$ because $f''(x)$ changes signs from $\ominus$ to $\oplus$.

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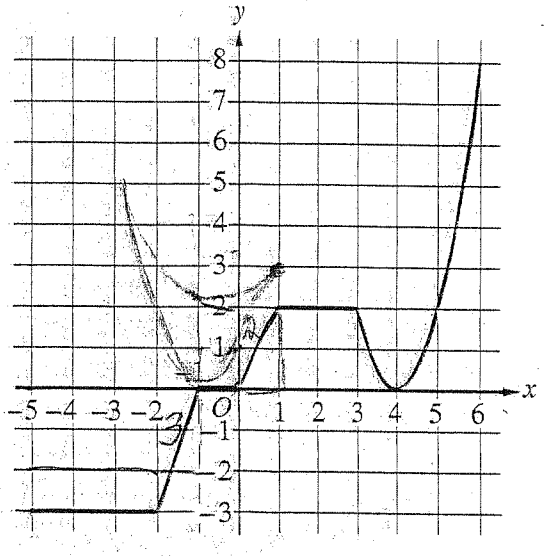

Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$f(-5) = 4$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$$\int_0^6 2(x-4)^2\,dx = 2x\,\frac{(x-4)^3}{3}\Big|_0^6 = 2(6)\left(\frac{(6-4)^3}{3}\right) - 0$$

$$= 12\left(\frac{8}{3}\right) = \frac{96}{3} = 32$$

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Continue question 3 on page 15.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

$f' > 0$ $f'' > 0$

$g(x) = 2(x-4)^2$ $\quad u = 2$ $\quad v = (x-4)^2$ $\quad 0 \geq x \geq 6$

$g'(x) = 2(2x-8) = 0$ $\quad u' = 0$ $\quad v' = 2(x-4)$

$4(x-4)$

$x = 4$

$-$ | $+$

4

f is increasing ($f' > 0$) and concave up ($f'' > 0$) at $4 \geq x \geq 6$

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

$g'(x) = f''(x) = 4x - 16 = 0$

$4(x-4)$

$x = 4$

Point of inflection at 4 because $f''(x) = g'(x)$ is changing signs

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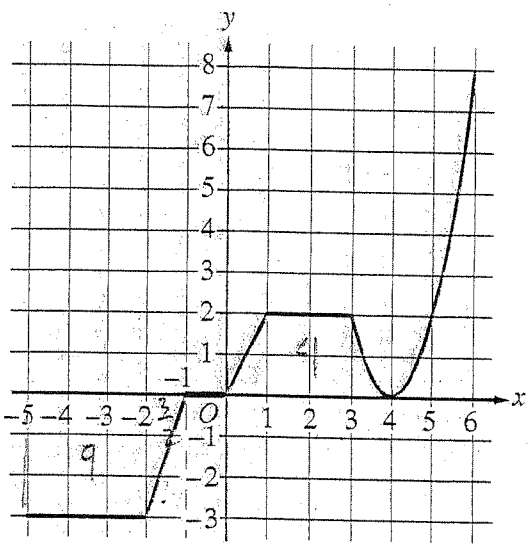

Graph of $g = f'$

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

$3 - \frac{3}{2} - 9$

$\frac{6}{2} - \frac{3}{2} - \frac{18}{2} = -\frac{15}{2}$

$f(-5) = -\frac{15}{2}$

(b) Evaluate $\int_1^6 g(x)\,dx$.

$g(x) = 2(x-4)^2$

$u = x - 4 \qquad du = dx$

$2\int u^2\,dx \qquad 2$

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Continue question 3 on page 15.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(0, 1) and (4, 6)

Those are intervals are increasing because they are above the x-axis and positive. The intervals are concave up because they are increasing. The intervals are both positive and increasing, thus both increasing and concave up.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

There are no point of inflection on the graph of f because a point of inflection on g is the point transfer from increasing to decreasing. On g, no ~~f~~ such point exist, instead, a horizontal linear line occurs.

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AP® SAMPLE STUDENT RESPONSES AND SCORING NOTES
2018 AP CALCULUS AB/CALCULUS BC

Question 3

Sample Identifier: A
Score: 9

- **The response earned 2-3-2-2.**
- In part (a) the response earned the first point with the expression $-\frac{1}{2}(1)(2)+\frac{1}{2}(3)(1)+3(3)$ which is $\int_{1}^{-5} g(x)dx$ evaluated geometrically. The response earned the second point with the expression $3-\frac{1}{2}(1)(2)+\frac{1}{2}(3)(1)+3(3)$ which added the initial value, 3, with the evaluation of the definite integral to compute $f(-5)$. Numeric expressions need not be simplified.
- In part (b) the response earned the first point with the expression $4+\int_{3}^{6} 2(x-4)^2\,dx$ because 4 is the value of $\int_{1}^{3} g(x)dx$. The response earned the second point with the expression for the antiderivative, $\frac{2(x-4)^3}{3}$, in the second line. The response earned the third point with the expression $4+\frac{2(2)^3}{3}+\frac{2}{3}$. Numeric expressions need not be simplified.
- In part (c) the response earned the first point with the intervals "(0, 1) (4, 6)". The response earned the second point with the reason "f' or g is > 0 and because f'' or g' is > 0."
- In part (d) the response earned the first point by identifying that a point of inflection occurs at $x=4$. The response earned the second point with the reason "f'' or g' changes from negative to positive".

Sample Identifier: B
Score: 8

- **The response earned 2-3-1-2.**
- In part (a) the response earned the first point with the expression $\int_{1}^{-5} g(x)dx$ in the second line. This response would have earned the second point by the numerical expression at the end of the second line, but responses that go on to simplify must do so correctly. This response simplifies correctly to $\frac{25}{2}$ earning the second point.
- In part (b) the response earned the first point with the expression $\int_{1}^{3} 2\,dx+\int_{3}^{6}(2(x-4)^2)\,dx$. This response earned the second point with the antiderivative expression $2(\frac{1}{3}x^3-4x^2+16x)$ in the second line. This response would have earned the third point by the numerical expression in the third line, but responses that go on to simplify must do so correctly. This response simplifies correctly to 10 earning the point.

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Question 3 (continued)

- In part (c) the response earned the first point with the intervals "$(0,1) \cup (4,6)$". This response did not earn the second point because a reason was not given. The sign chart alone does not earn the second point; the response must identify the relevant information in the sign chart.
- In part (d) the response earned the first point by identifying that a point of inflection occurs at $x = 4$. The response earned the second point by stating the connection $g'(x) = f''(x)$ above the sign chart and giving the reason "a change in sign of $f''(x)$". To use a reason with only f'', the connection $g'(x) = f''(x)$ was required.

Sample Identifier: C
Score: 7

- **The response earned 1-2-2-2.**
- In part (a) the response earned the first point with the expression $\int_{-5}^{1} f'(x)dx$ in the first line. This response had an error when evaluating the definite integral in the numeric expression on the third line resulting in an incorrect answer. The error was omitting $\frac{1}{2} \cdot 1 \cdot 2$. This response, therefore, did not earn the second point.
- In part (b) the response earned the first point with the expression $\int_{1}^{3} g(x)\,dx + \int_{3}^{6} (2(x-4)^2)\,dx$. This response earned the second point with the antiderivative expression $2\left(\frac{1}{3}\right)(x-4)^3$ in the third line. There is an error evaluating $\int_{1}^{3} g(x)dx$ in the third line resulting in an incorrect answer. This response, therefore, did not earn the third point.
- In part (c) the response earned the first point with the intervals "$(0,1) \cup (4,6)$". The response earned the second point with the reason "on these intervals $f' = g$ is positive and increasing".
- In part (d) the response earned the first point by identifying that a point of inflection occurs at $x = 4$. The response earned the second point with the reason "$f' = g$ changes from decreasing to increasing at $x = 4$".

Sample Identifier: D
Score: 6

- **The response earned 2-0-2-2.**
- In part (a) the response earned the first point with the expression $\int_{1}^{-5} g(x)dx$ in the first line. This response would have earned the second point by the first numerical expression in the second line, but responses that go on to simplify must do so correctly. This response simplifies correctly to 12.5 earning the second point.
- In part (b) the response did not earn the first point because the integral is never written as the sum of two integrals split at $x = 3$ or the equivalent. Because an antiderivative was never computed,

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Question 3 (continued)

the response fails to earn the second point. As a result of this error, the response was ineligible for the answer point.

- In part (c) the response earned the first point with the intervals "$(0,1)$ & $(4,6)$". The response earned the second point with the reason "the value & slope of g are both positive".
- In part (d) the response earned the first point by identifying that a point of inflection occurs at $x = 4$. The response earned the second point with the reason that the slope of g switching from negative to positive.

Sample Identifier: E
Score: 5

- **The response earned 1-1-2-1.**
- In part (a) the response earned the first point with the expression $\int_{1}^{-5} f'(x)dx$ in the first line. This response would have earned the second point by the numerical expression in the second line, but responses that go on to simplify must do so correctly. This response simplifies incorrectly to -6.5 and did not earn the second point.
- In part (b) the response did not earn the first point because $\frac{(2)(2)}{2}$ is not the value of $\int_{1}^{3} g(x)\,dx$. The correct value of $\int_{1}^{3} g(x)\,dx$ summed with $\int_{3}^{6} 2(x-4)^2\,dx$ was required when $\int_{1}^{3} g(x)\,dx$ was not symbolically given in the integral expression. The response earned the second point with the antiderivative expression on the second line. The response did not earn the third point because the answer 8 was not the correct answer 10.
- In part (c) the response earned the first point with the intervals "$(0,1) \cup (4,6)$". The response earned the second point with the reason "$(g > 0)$ & $(g' > 0)$".
- In part (d) the response earned the first point by identifying that a point of inflection occurs at $x = 4$. The reason "the slope, g, from the left is negative and from the right is positive" is incorrect and the second point was not earned.

Sample Identifier: F
Score: 4

- **The response earned 0-2-1-1.**
- In part (a) the response did not earn the first point. An integral expression was never presented, nor its equivalent numeric expression. The value given for $f(-5)$ is incorrect so the second point was not earned.
- In part (b) the response earned the first point with the expression $4 + \int_{3}^{6} 2(x-4)^2\,dx$ because 4 is the value of $\int_{1}^{3} g(x)\,dx$. This response earned the second point with the antiderivative expression

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Question 3 (continued)

$\frac{2}{3}(x-4)^3$ in the second line. This response would have earned the third point by the first numerical expression in the second line, but responses that go on to simplify must do so correctly. This response simplifies incorrectly to 13 and did not earn the third point.

- In part (c) the response earned the first point with the intervals $(0,1)$ and $(4,6)$. The statement "the graph of f is both positive and increasing" is correct, but does not explain why f is increasing and concave up. A valid reason was not given and so this response did not earn the second point.
- In part (d) the response earned the first point by identifying that a point of inflection occurs at $x = 4$. The reason "the graph changes sign" is too vague because the graph of g is given in the stem of the problem and part (d) asks above the graph of f. This response did not earn the second point.

Sample Identifier: G
Score: 3

- **The response earned 0-1-1-1.**
- In part (a) the response did not earn the first point. An integral expression was never presented, nor its equivalent numeric expression. The value given for $f(-5)$ is incorrect so the second point was not earned.
- In part (b) the response did not earn the first point because $\int_1^6 g(x)dx$ is never written as the sum of two integrals split at $x = 3$ or the equivalent. This response earned the second point with the antiderivative expression $\frac{2}{3}(x-4)^3$ in the second line. The answer $\frac{70}{3}$ is not correct so this response did not earn the third point.
- In part (c) the response earned the first point with the intervals $(0,1)$ and $(4,6)$. $f''(x) > 0$ cannot be used in the reason without explicitly stating the connection $g'(x) = f''(x)$. This response does not state that explicit connection and did not earn the second point.
- In part (d) the response earned the first point by identifying that a point of inflection occurs at $x = 4$. The second point is not earned because $f''(x)$ changes signs cannot be used in the reason without explicitly stating the connection $g'(x) = f''(x)$. This response does not state that explicit connection.

Sample Identifier: H
Score: 2

- **The response earned 0-0-0-2.**
- In part (a) the response did not earn the first point. An integral expression was never presented, nor its equivalent numeric expression. The value given for $f(-5)$ is incorrect so the second point was not earned.
- In part (b) the response did not earn the first point because $\int_1^6 g(x)dx$ is never written as the sum of two integrals split at $x = 3$ or the equivalent. The antiderivative expression on the second line is

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Question 3 (continued)

incorrect so the second point was not earned. As a result of this error, the response was ineligible for the third point.

- In part (c) the response did not earn the first point. The interval $4 \geq x \geq 6$ is incorrect so this response did not earn the second point. If the response has one interval, it must be one of the intervals $0 < x < 1$ or $4 < x < 6$ to be eligible for the reason point. This response does not meet that criteria and is ineligible for the second point.
- In part (d) the response earned the first point by identifying that a point of inflection occurs at $x = 4$. The response earned the second point with the reason "$f''(x) = g'(x)$ is changing signs".

Sample Identifier: I
Score: 1

- **The response earned 0-0-1-0.**
- In part (a) the response did not earn the first point. An integral expression was never presented, nor its equivalent numeric expression. The value given for $f(-5)$ is incorrect so the second point was not earned.
- In part (b) the response did not earn the first point because $\int_1^6 g(x)dx$ is never written as the sum of two integrals split at $x = 3$ or the equivalent. The antiderivative of $2(x-4)^2$ was never presented and so the response did not earn the second point. Without the antiderivative point, the response was ineligible for the third point.
- In part (c) the response earned the first point with the intervals $(0,1)$ and $(4,6)$. In the reason given, the response did not identify which function was positive and increasing using the vague language "they". Consequently, the response did not earn the second point.
- In part (d) the response did not earn the first point because $x = 4$ was not identified as the *x*-coordinate of the point of inflection. Without identifying this *x*-coordinate, the response was ineligible for the second point.

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

   (a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

   (b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

   (c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

   (d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

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Question 4

(a) $H'(6) \approx \dfrac{H(7) - H(5)}{7 - 5} = \dfrac{11 - 6}{2} = \dfrac{5}{2}$

$H'(6)$ is the rate at which the height of the tree is changing, in meters per year, at time $t = 6$ years.

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\dfrac{H(5) - H(3)}{5 - 3} = \dfrac{6 - 2}{2} = 2$

Because H is differentiable on $3 \le t \le 5$, H is continuous on $3 \le t \le 5$.

By the Mean Value Theorem, there exists a value c, $3 < c < 5$, such that $H'(c) = 2$.

$2 : \begin{cases} 1 : \dfrac{H(5) - H(3)}{5 - 3} \\ 1 : \text{conclusion using Mean Value Theorem} \end{cases}$

(c) The average height of the tree over the time interval $2 \le t \le 10$ is given by $\dfrac{1}{10 - 2}\displaystyle\int_2^{10} H(t)\, dt$.

$$\frac{1}{8}\int_2^{10} H(t)\, dt \approx \frac{1}{8}\left(\frac{1.5 + 2}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot 3\right)$$
$$= \frac{1}{8}(65.75) = \frac{263}{32}$$

The average height of the tree over the time interval $2 \le t \le 10$ is $\dfrac{263}{32}$ meters.

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \end{cases}$

(d) $G(x) = 50 \Rightarrow x = 1$

$$\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1 + x)100 - 100x \cdot 1}{(1 + x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1 + x)^2} \cdot \frac{dx}{dt}$$

$$\left.\frac{d}{dt}(G(x))\right|_{x=1} = \frac{100}{(1 + 1)^2} \cdot 0.03 = \frac{3}{4}$$

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is $\dfrac{3}{4}$ meter per year.

$3 : \begin{cases} 2 : \dfrac{d}{dt}(G(x)) \\ 1 : \text{answer} \end{cases}$

Note: max $1/3$ [1-0] if no chain rule

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$H'(6) \approx \frac{H(7)-H(5)}{7-5} = \frac{11-6}{7-5} = \frac{5}{2}$

This means that the tree is growing at a rate of approximately 2.5 meters per year at the 6-year mark

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

There must be at least one point where $H'(t) = 2$ because by the mean value theorem, $H'(t) = \frac{H(5)-H(3)}{5-3} = \frac{6-2}{2} = 2$

Since the values of $t=3$ and $t=5$ are on the interval $2<t<10$ and the mean value theorem states that $H'(t)$ will equal 2 between $t=3$ and $t=5$, then this implies $H'(t)=2$ somewhere on $2<t<10$ as well.

The mean value theorem is applicable here because $H(t)$ is continuous on $[2,10]$ and differentiable on $(2,10)$

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Continue question 4 on page 17.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

$$\int_2^{10} H(t)\,dt \approx \frac{1}{2}(1)(1.5+2) + \frac{1}{2}(2)(2+6) + \frac{1}{2}(7)(6+11) + \frac{1}{2}(3)(11+15)$$

$$= \frac{7}{4} + 8 + 17 + 34 = \frac{263}{4}$$

$$\frac{\int_2^{10} H(t)\,dt}{10-2} = \frac{\frac{263}{4}}{8} = \frac{263}{32}$$ meters is the average height of the tree

---

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$dx/dt = 0.03$ m/year

$$G'(x) = \frac{(1+x)100\frac{dx}{dt} - 100x\frac{dx}{dt}}{(1+x)^2} = \frac{100\frac{dx}{dt} - 100x\frac{dx}{dt} - 100x\frac{dx}{dt}}{(1+x)^2} = \frac{100\frac{dx}{dt}}{(1+x)^2}$$

diameter when $G(x)=50$

$$50 = \frac{100x}{1+x} \rightarrow 50+50x = 100x$$
$$50x = 50$$
$$x = 1 \text{ meter}$$

$$G'(x)\Big|_{x=50} = \frac{100(0.03)}{(1+1)^2} = \frac{3}{4}$$

When the height of the tree is 50 meters the rate of change of the height is $\frac{3}{4}$ meters per year

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$H'(6) \approx \frac{11-6}{7-5} = \frac{5}{2}$

$H'(6) \approx 2.5$ meters per year

$H'(6)$ represents the rate of change of the height of the tree when $t=6$. In other words, it is the rate at which the tree is growing when the tree is 6 years old.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

$H(t)$ is a continuous, differentiable function over $(2, 10)$.

$H(5)=6$ and $H(3)=2$, so $\frac{H(5)-H(3)}{5-3} = \frac{6-2}{5-3} = \frac{4}{2} = 2$

Thus by the Mean Value Theorem, there must be at least one time t in the interval $[3, 5]$ such that $H'(t)=2$. Since $[3, 5]$ lies completely within $(2, 10)$, t must lie within $(2, 10)$ as well.

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Continue question 4 on page 17.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

$H_{ave} \approx \frac{1}{10-2} \cdot \frac{1}{2}\left[(2+1.5)(1) + (6+2)(2) + (11+6)(2) + (15+11)(3)\right]$

$H_{ave} \approx \frac{1}{16}\left[3.5 + 16 + 34 + 78\right]$

$H_{ave} \approx \frac{1}{16}\left[131.5\right]$

$H_{ave} \approx \frac{263}{32}$ meters

The average height of the tree was $\frac{263}{32}$ meters over $2 \le t \le 10$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$\frac{dx}{dt} = 0.03 \quad G(x) = 50 = \frac{100x}{1+x}$

$50 + 50x = 100x$

$50x = 50$

$x = 1$

$G = \frac{100x}{1+x}$

$\frac{dG}{dt} = \frac{100(1+x)\cdot\frac{dx}{dt} - 100x\cdot\frac{dx}{dt}}{(1+x)^2}$

$\frac{dG}{dt} = \frac{100(1+1)\cdot 0.03 - 100(1)(0.03)}{(1+1)^2}$

$\frac{dG}{dt} = \frac{0.03}{4}$

$\frac{dG}{dt} = 0.0075$ meters per year

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B

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$\frac{11-6}{7-5} = \frac{5}{2}$ m/year → avg rate of change of the tree's height in meters from 5 to 7 years or at 6 years

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

From (3, 5) the average slope is $\frac{6-2}{5-3} = \frac{4}{2} = 2$

mean value theorem $H'(t)$ = avg slope somewhere

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Continue question 4 on page 17.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

$\frac{1}{8}\int_2^{10} H(t)\,dt$ $\quad \frac{1}{8}\left[\frac{1}{2}(1)(1.5+2) + \frac{1}{2}(2)(2+6) + \frac{1}{2}(2)(6+11) + \frac{1}{2}(3)(11+15)\right]$

$\frac{1}{8}\left[\frac{3.5}{2} + \frac{16}{2} + \frac{34}{2} + \frac{78}{2}\right]$

$\frac{1}{8}\frac{131.5}{2}$ $\quad \frac{131.5}{16}$ $\quad \boxed{\frac{263}{32}}$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$G'(x) = \frac{100(1+x)\frac{dx}{dt} - 100x\frac{dx}{dt}}{(1+x)^2}$

$50 = \frac{100x}{1+x}$

$50 + 50x = 100x$

$50 = 50x$

$x = 1$ at 50 meters

$\frac{100(1+1)(.03) - 100(1)(.03)}{(1+1)^2}$

$\frac{(200)(.03) - 100(.03)}{4}$

$\frac{100(.03)}{4}$ $\quad 25(.03) = .75$ m/year

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$$\frac{H(7)-H(5)}{7-5} = \frac{11-6}{7-5} = \frac{5}{2} \text{ meters/year}$$

$H'(6)$ is the rate of change of the height of the tree in terms of years.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

$$\frac{H(5)-H(3)}{5-3} = \frac{6-2}{5-3} = \frac{4}{2} = 2$$

There must be a time such that $H'(t)=2$ because based on the mean value theorem, since the average slope/rate of change between 3 and 5 is 2 meters/year, there must be another point that also shares the same rate of change in between.

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Continue question 4 on page 17.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

$\frac{(a+b)}{2}h$

$\frac{(1.5+2)}{2}(1) + \frac{(2+6)}{2}(2) + \frac{(6+11)}{2}(2) + \frac{(11+15)}{2}(3)$

$\frac{3.5}{2} + 8 + 17 + (13)(3)$

$1.75 + 25 + 39$

$= 64 + 1.75 = \underline{65.75 \text{ meters}}$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$50 = \frac{100x}{1+x}$

$50 + 50x = 100x$

$50 = 50x$

$x = 1$

$G(x) = \frac{100x}{1+x}$

$\frac{dG}{dt} = \frac{(1+x)(100) - (100x)}{(1+x)^2}\frac{dx}{dt}$

$\frac{dG}{dt} = \frac{(1+1)(100) - (100)(1)}{(1+1)^2}(.03)$

$\frac{dG}{dt} = \frac{100}{4}(.03)$

$\frac{dG}{dt} = 25(.03) = \frac{75}{100}$ m/year

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$$\frac{H(7)-H(5)}{7-5}$$

$$\frac{11-6}{7-5} = \frac{5}{2} = 2.5$$

$$H'(6) = 2.5$$

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

$$\frac{H(5)-H(3)}{5-3}$$

$$\frac{6-2}{5-3} = \frac{4}{2} = 2$$

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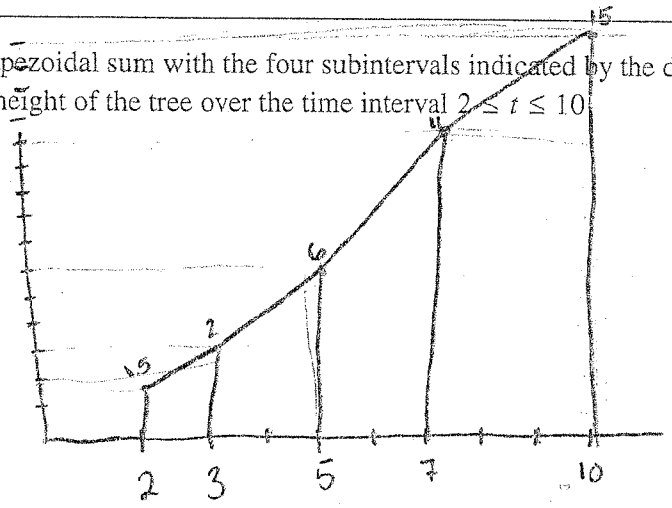
Continue question 4 on page 17.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.



16.137
4 | 65.75
-4
25
24
17
16
15

$\frac{(2+1.5)}{2} \cdot 1 + \frac{(6+2)}{2} \cdot 2 + \frac{(11+6)}{2} \cdot 2 + \frac{(15+11)}{2} \cdot 3$

$1.75 + 8 + 17 + 39$

$1.75 + 8 + 56$

$\frac{65.75}{4}$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$$G'(x) = \frac{(1+x)100\frac{dx}{dt} - 100x\frac{dx}{dt}}{(1+x)^2}$$

$$= \frac{3(1+x) - 3x}{(1+x)^2}$$

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$\frac{11-6}{7-5} = \frac{5}{2} = 2.5$ meters / years

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

Because the tree is growing, which there must be a time measurement.

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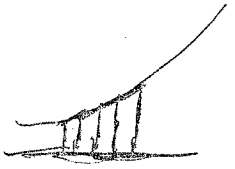
Continue question 4 on page 17.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.



$$\frac{(2+1.5)\times 1 + (2+6)\times 2 + (6+11)\times 2 + (15+11)\times 3}{2}$$

(3.5, 8, 17, 26, 13)

$= 3.5+8+17+39$

$= 67.5$

$67.5 \div 4 = \boxed{16.8}$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$$\frac{dG}{dt} = \frac{100(1+x)\frac{dx}{dt} - 100x\frac{dx}{dt}}{(1+x)^2}$$

$=$

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E

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$H'(6) = \frac{H(7) - H(5)}{7-5} = \frac{11-6}{2} = \frac{5}{2}$ or 2.5 m/year

$H'(6)$ is equivalent to the rate at which the tree is changing in height at time $t = 6$.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

Since $H(t)$ is continuous and differentiable on the interval $[2, 10]$, then by the Intermediate Value Theorem there must be one time t such that $H'(t) = 2$.

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Continue question 4 on page 17.

E

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

$\frac{1}{2}(2-1.5) + \frac{1}{2}(6-2) + \frac{1}{2}(11-6) + \frac{1}{2}(15-11)$

$\frac{1}{4} + 2 + \frac{5}{2} + 2$

$4 + \frac{1}{4} + \frac{5}{2}$ $\frac{10}{4}$

$4 + \frac{11}{4}$

$\frac{16}{4} + \frac{11}{4} = \boxed{\frac{27}{4}}$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$G'(x) = \frac{(1+x)(100) - (100x)(1)}{(1+x)^2}$ $G'(x) = 0.03$ when $x = 50$

$= \frac{100 + 100x - 100x}{(1+x)^2}$

$y = 100x$
$y' = 100$
$v = 1+x$
$v' = 1$

$\frac{100}{(1+50)^2} = \frac{100}{51^2}$

51×51
51
$+2550$
2601

$\boxed{\frac{100}{2601} \text{ m/year}}$

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	t (years)	2	3	5	7	10
	$H(t)$ (meters)	1.5	2	6	11	15

11:25
11:07

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$$\frac{11-6}{7-5} = H'(6)$$

t years
H(t) meters

H'(6) Means an estimate of the height of a tree at time t=6 years.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

There MUST be atleast one time t, for $2 < t < 10$, such that $H'(t) = 2$ because of the meanvalue theorm.

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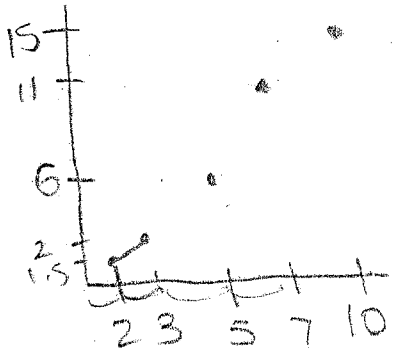
Continue question 4 on page 17.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.



$1 \times \frac{1}{2}(1.5+2)$
$2 \times \frac{1}{2}(2+6)$
$2 \times \frac{1}{2}(6+11)$
$3 \times \frac{1}{2}(11+15)$

$\left(1 \times \frac{1}{2}(1.5+2)\right) + \left(2 \times \frac{1}{2}(2+6)\right) + \left(2 \times \frac{1}{2}(6+11)\right) + \left(3 \times \frac{1}{2}(11+15)\right)$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$\int_0^{50} (G(x) - .03)\,dx$

$\frac{100x}{1+x} - .03x \Big|_0^{50}$

$\left(\frac{100 \cdot 50}{1+50} - .03 \cdot 50\right) - 0$

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NO CALCULATOR ALLOWED

J

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

H'(6) ~~units would be the am~~ meters/year/year. The derivative of h is the measure of ~~how~~ ~~fa~~ change in growth rate in a given year.

(b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

The interval measures from 2<t<10, therefore a point of H'(t)=2 must exist.

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Continue question 4 on page 17.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

$\frac{1}{2}\left(\frac{15-1.5}{4}\right)(1.5 + 2(2) + 2(6) + 2(11) + 15) =$

$= \frac{1}{2}(3.375)(1.5+4+12+22+15) = 1.6875(54.5)$

1.6875×54.5

84375

$+6750000$

8437500

91.96875

91.969 meters

Scratch work

$15 - 1.5 = 13.5$

$13.5 \div 4 = 3.375$

$3.375 \div 2 = 1.6875$

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

$$G'(x) = \frac{(1+x)(100) - (100x)(1)}{(1+x)^2}$$

$$= \frac{100 + 100x - 100x}{x^2+2x+1}$$

$$G'(50) = \frac{100}{(50)^2+2(50)+1} = \frac{100}{2500+100+1} = \boxed{\frac{100}{2601} \frac{m}{y}}$$

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AP® SAMPLE STUDENT RESPONSES AND SCORING NOTES
2018 AP CALCULUS AB/CALCULUS BC

Question 4

Sample Identifier: A
Score: 9

- **The response earned 2-2-2-3.**
- In part (a) the response would have earned the first point for the correct, un-simplified estimate of $\frac{11-6}{7-5}$. The estimate is correctly simplified to $\frac{5}{2}$ which earned the point. The response earned the second point for the interpretation that included the three necessary elements: an interpretation of H' as a rate in the context of the tree, correct units of meters per year, and an interpretation of the input "6" as the moment in time of year 6.
- In part (b) the response earned the first point for the difference quotient $\frac{H(5)-H(3)}{5-3}$. The first point does not require the substitution of function values and simplification that follows; this work was considered in the context of the second point. The second point was earned for the explanation using the secant slope of 2 found from the difference quotient; the response explicitly states that "$H(t)$ is continuous" at the end of the paragraph, which was necessary to earn the point. While not required, the response references the Mean Value by name, explains that the values of $t = 3$ and $t = 5$ are on the interval (2, 10) and also explicitly states the differentiability of H. The additional information in the response is correct.
- In part (c) the response earned the first point for presenting the correct setup for the trapezoidal sum in the first line. The arithmetic and simplification that follows is considered for the second point. The response would have earned the second point for presenting the approximation $\frac{\frac{263}{4}}{8}$. The approximation is correctly simplified which earned the point. The response includes a sentence that explains the numerical answer in context. This interpretation was not required to earn the second point.
- In part (d) the response earned the first two points with the correct derivative shown in the first line: $G'(x) = \frac{(1+x)100\frac{dx}{dt} - 100x\frac{dx}{dt}}{(1+x)^2}$. The simplification of this expression which follows is considered for the third point. The response would have earned the third point with the un-simplified answer $\frac{100(0.03)}{(1+1)^2}$. This answer was correctly simplified to $\frac{3}{4}$ which earned the third point. The response includes a sentence that explains the numerical answer in context. This interpretation was not required to earn the third point.

Sample Identifier: G
Score: 8

- **The response earned 2-2-2-2.**
- In part (a) the response would have earned the first point for the correct, un-simplified estimate of $\frac{11-6}{7-5}$. The estimate is correctly simplified to 2.5 which earned the point. While not required for the first point, the answer includes the correct units of meters/year which was considered for the second point. The response earned the second point for the interpretation that included the three

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Question 4 (continued)

necessary elements: an interpretation of H' as a rate in the context of the tree, correct units of meters per year (given with the estimate of 2.5), and an interpretation of the input "6" as the moment in time of year 6.

- In part (b) the response earned the first point for the difference quotient $\frac{H(5)-H(3)}{5-3}$. The first point does not require the substitution of function values and simplification that follows; this work was considered in the context of the second point. The second point was earned for stating that $H(t)$ is continuous, seen in the first line, and stating "there must be at least one time t in the interval…such that $H'(t)=2$." While not required, there is additional explanation. This additional explanation is correct.
- In part (c) the response earned the first point for the un-simplified expression $\frac{1}{2}[(2+1.5)(1)+(6+2)(2)+(11+6)(2)+(15+11)(3)]$, seen in the first line. The arithmetic and simplification that follows is considered for the second point. The response would have earned the second point for multiplying the correct trapezoidal sum by $\frac{1}{10-2}$, shown in the first line. The approximation was correctly simplified to $\frac{263}{32}$ which earned the second point. The response includes units and an interpretation sentence which were not required to earn the second point.
- In part (d) the response earned both of the first two points for the expression $\frac{dG}{dt}=\frac{100(1+x)\cdot\frac{dx}{dt}-100x\cdot\frac{dx}{dt}}{(1+x)^2}$; this expression includes the chain rule and a correct quotient rule derivative. The response would have earned the third point for the un-simplified answer of $\frac{100(1+1)\cdot 0.03-100(1)(0.03)}{(1+1)^2}$. The answer was simplified incorrectly and does not earn the third point.

Sample Identifier: B
Score: 7

- **The response earned 1-1-2-3.**
- In part (a) the response would have earned the first point for the correct, un-simplified estimate of $\frac{11-6}{7-5}$. The estimate is correctly simplified to 2.5 which earned the point. The response did not earn the second point, as the interpretation of $H'(6)$ was incorrectly given as the average rate of change over an interval.
- In part (b) the response earned the first point for the difference quotient $\frac{6-2}{5-3}$. The simplification of this expression to a value of 2 was considered in the context of the second point. The second point was not earned because the response did not discuss the continuity of *H*. This necessary condition for the application of the Mean Value Theorem was not explicitly stated in the question and so needed to be declared in the response to earn the second point.

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Question 4 (continued)

- In part (c) the response earned the first point for the un-simplified expression $\frac{1}{2}(1)(1.5+2)+\frac{1}{2}(2)(2+6)+\frac{1}{2}(2)(6+11)+\frac{1}{2}(3)(11+15)$ in brackets on the first line. The arithmetic and simplification that follows is considered for the second point. The response would have earned the second point for the expression in the first line that includes the factor of $\frac{1}{8}$ attached to the correct (un-simplified) trapezoidal sum. The approximation was correctly simplified to $\frac{263}{32}$ which earned the second point.
- In part (d) the response earned the first two points with the correct derivative shown in the first line: $G'(x)=\frac{100(1+x)\frac{dx}{dt}-100x\frac{dx}{dt}}{(1+x)^2}$. The response would have earned the third point with the display of the un-simplified answer $\frac{100(1+1)(.03)-100(1)(.03)}{(1+1)^2}$. This answer was correctly simplified to 0.75 which earned the third point. The response includes units which were not required to earn the third point.

Sample Identifier: C
Score: 6

- **The response earned 1-1-1-3.**
- In part (a) the response would have earned the first point for the correct, un-simplified estimate of $\frac{11-6}{7-5}$. The estimate is correctly simplified to $\frac{5}{2}$ which earned the point. While not required for the first point, the answer includes the correct units of meters/year which was considered for the second point. The response did not earn the second point, as the explanation of $H'(6)$ does not include the interpretation of the input value of 6 as the moment in time of 6 years.
- In part (b) the response earned the first point for the difference quotient $\frac{H(5)-H(3)}{5-3}$. The first point does not require the substitution of function values and simplification that follows; this work was considered in the context of the second point. The second point was not earned because, although the Mean Value Theorem is explained in the context of our problem, the response does not state the necessary condition that *H* is continuous; to earn the second point, the response must conclude that *H* is continuous from the given information that *H* is twice-differentiable.
- In part (c) the response earned the first point for the un-simplified expression seen in the second line, below the general formula for the area of the trapezoid. The arithmetic and simplification that follows is considered for the second point. The response did not earn the second point because the response did not multiply by $\frac{1}{8}$ to find the average value of the function $H(t)$ on the interval $[2,10]$.
- In part (d) the response earned the first two points with the correct derivative shown in the second line of work on the right hand side. This derivative includes both the correct quotient rule and the chain rule. The response would have earned the third point with the un-simplified answer

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Question 4 (continued)

$\frac{(1+1)(100)-(100)(1)}{(1+1)^2}(.03)$. This answer is correctly simplified to $\frac{75}{100}$ which earned the third point. The response included units which were not required to earn the third point.

Sample Identifier: D
Score: 5

- **The response earned 1-1-1-2.**
- In part (a) the response would have earned the first point for the correct, un-simplified estimate of $\frac{11-6}{7-5}$. The estimate is correctly simplified to 2.5 which earned the point. The response did not earn the second point, as there is no interpretation of $H'(6)$.
- In part (b) the response earned the first point for the difference quotient $\frac{H(5)-H(3)}{5-3}$. The first point does not require the substitution of function values and simplification that follows; this work was considered in the context of the second point. The second point was not earned because no explanation was given.
- In part (c) the response earned the first point for the un-simplified expression seen immediately below the graph showing trapezoids. The arithmetic and simplification that follows is considered for the second point. The response did not earn the second point because the response divided by 4 instead of by 8 in an attempt to find the average value of the function $H(t)$ on the interval [2, 10].
- In part (d) the response earned the first two points with the correct derivative shown in the first line of work. This derivative includes both the correct quotient rule and the chain rule. The response began substituting values into this correct derivative but did not substitute a value for the diameter *x* and so the response did not earn the third point.

Sample Identifier: F
Score: 4

- **The response earned 1-0-1-2.**
- In part (a) the response would have earned the first point for the correct, un-simplified estimate of $\frac{11-6}{7-5}$. The estimate is correctly simplified to 2.5 which earned the point. While not required for the first point, the answer includes the correct units of meters/year which was considered for the second point. The response did not earn the second point, as there is no interpretation of $H'(6)$. The units of meters/year do not provide a complete interpretation: the response must also reference the rate in the context of the tree and the moment in time of $t = 6$.
- In part (b) the response did not earn either point as there is no difference quotient given. Additionally, the explanation does not provide any mathematical reasoning.
- In part (c) the response earned the first point for the un-simplified expression seen in the first line, showing correct trapezoidal areas for the four subintervals given in the table. The arithmetic that follows is considered for the second point. There is an arithmetic mistake in the first line and the response incorrectly divided by 4 in order to find the average value. Thus, the second point was not earned.

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Question 4 (continued)

- In part (d) the response earned the first two points with the correct derivative shown. This derivative includes both the correct quotient rule and the chain rule. The response did not substitute any values to obtain an answer and so did not earn the third point.

Sample Identifier: E
Score: 3

- **The response earned 2-0-0-1.**
- In part (a) the response would have earned the first point for the correct, un-simplified estimate of $\frac{11-6}{2}$. This estimate is correctly simplified to "$\frac{5}{2}$ or 2.5" which earned the first point. While not required for the first point, the answer includes the correct units of meters/year which was considered for the second point. The response earned the second point for the interpretation that included the three necessary elements: an interpretation of H' as a rate in the context of the tree, correct units of meters per year (given with the estimate), and an interpretation of the input "6" as the moment in time of $t = 6$.
- In part (b) the response did not earn the first point because no difference quotient is given. The response is not eligible for the second point because the explanation given does not reference the interval [3, 5] that results in the secant slope of 2. Additionally, the second point could not be earned because the wrong theorem is cited.
- In part (c) the response did not earn the first point because correct trapezoid areas are not given. The response did not earn the second point because the correct approximation is not given.
- In part (d) the response earned one of the first two points for the correct derivative (with respect to x) given in the first line. As indicated on the scoring guideline, a derivative that does not include the chain rule is not eligible for any additional points.

Sample Identifier: H
Score: 2

- **The response earned 1-0-1-0.**
- In part (a) the response earned the first point for the correct estimate $\frac{11-6}{7-5}$. The response did not earn the second point because $H'(6)$ is incorrectly interpreted to be an estimate of the height of the tree, not a rate. Additionally, no units are included.
- In part (b) the response did not earn the first point because no difference quotient is given. The response is not eligible for the second point because the explanation given does not reference the interval [3, 5] that results in the secant slope of 2.
- In part (c) the response earned the first point for the correct trapezoidal sum expression written in the box. The second point is not earned because the response does not multiply by $\frac{1}{8}$ to find the average value of the function $H(t)$ on the interval [2, 10].
- In part (d) the response did not earn any points because there is no attempt to find a derivative of the function $G(x)$. Note: crossed off work, as seen in the first line, is not read for any points.

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AP® SAMPLE STUDENT RESPONSES AND SCORING NOTES
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Question 4 (continued)

Sample Identifier: J
Score: 1

- **The response earned 0-0-0-1.**
- In part (a) the response did not earn the first point because no estimate is given. The response did not earn the second point because the interpretation is incorrect: the derivative is not the change in growth rate, the units are incorrect, and the moment in time of $t = 6$ is not referenced.
- In part (b) the response did not earn the first point because no difference quotient is given. The response did not earn the second point because the explanation is repeating the question without providing mathematical reasoning connected to the table given.
- In part (c) the response did not earn the first point because correct trapezoidal areas are not given. The response appears to be attempting to use a "trapezoidal rule" that would be appropriate if the subintervals in the table were of equal length. The response did not earn the second point because, without correct trapezoidal areas, the correct answer could not be found.
- In part (d) the response earned one of the first two points for the correct derivative (with respect to x) that is given in the first line. Note: the antiderivative $x + x^2$ has been crossed out and replaced with 1. As indicated on the scoring guideline, a derivative that does not include the chain rule is not eligible for any additional points.

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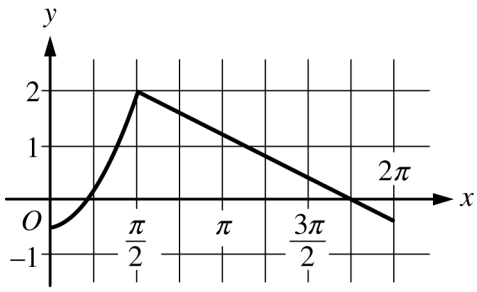
5. Let f be the function defined by $f(x) = e^x \cos x$.

   (a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

   (b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

   (c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

   (d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

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AP® CALCULUS AB
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Question 5

(a) The average rate of change of f on the interval $0 \le x \le \pi$ is $\dfrac{f(\pi) - f(0)}{\pi - 0} = \dfrac{-e^{\pi} - 1}{\pi}$.

1 : answer

(b) $f'(x) = e^x \cos x - e^x \sin x$

$f'\left(\dfrac{3\pi}{2}\right) = e^{3\pi/2}\cos\left(\dfrac{3\pi}{2}\right) - e^{3\pi/2}\sin\left(\dfrac{3\pi}{2}\right) = e^{3\pi/2}$

The slope of the line tangent to the graph of f at $x = \dfrac{3\pi}{2}$ is $e^{3\pi/2}$.

2 : { 1 : $f'(x)$; 1 : slope }

(c) $f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \dfrac{\pi}{4}, x = \dfrac{5\pi}{4}$

x	$f(x)$
0	1
$\dfrac{\pi}{4}$	$\dfrac{1}{\sqrt{2}}e^{\pi/4}$
$\dfrac{5\pi}{4}$	$-\dfrac{1}{\sqrt{2}}e^{5\pi/4}$
2π	$e^{2\pi}$

The absolute minimum value of f on $0 \le x \le 2\pi$ is $-\dfrac{1}{\sqrt{2}}e^{5\pi/4}$.

3 : { 1 : sets $f'(x) = 0$; 1 : identifies $x = \dfrac{\pi}{4}, x = \dfrac{5\pi}{4}$ as candidates ; 1 : answer with justification }

(d) $\lim\limits_{x \to \pi/2} f(x) = 0$

Because g is differentiable, g is continuous.

$\lim\limits_{x \to \pi/2} g(x) = g\left(\dfrac{\pi}{2}\right) = 0$

By L'Hospital's Rule,

$\lim\limits_{x \to \pi/2} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to \pi/2} \dfrac{f'(x)}{g'(x)} = \dfrac{-e^{\pi/2}}{2}$.

3 : { 1 : g is continuous at $x = \dfrac{\pi}{2}$ and limits equal 0 ; 1 : applies L'Hospital's Rule ; 1 : answer }

Note: max $1/3$ [1-0-0] if no limit notation attached to a ratio of derivatives

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N

5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

average $f' = \dfrac{f(\pi) - f(0)}{\pi - 0} = \dfrac{-e^{\pi} - 1}{\pi}$

(b) What is the slope of the line tangent to the graph of f at $x = \dfrac{3\pi}{2}$?

$f'(x) = e^x \cdot \cos x + e^x \cdot (-\sin x) = e^x(\cos x - \sin x)$

$f'\left(\dfrac{3\pi}{2}\right) = e^{\frac{3\pi}{2}} \cdot (0 + 1) = e^{\frac{3\pi}{2}}$

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Continue question 5 on page 19.

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

$f'(x) = 0 = e^x(\cos x - \sin x) \rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

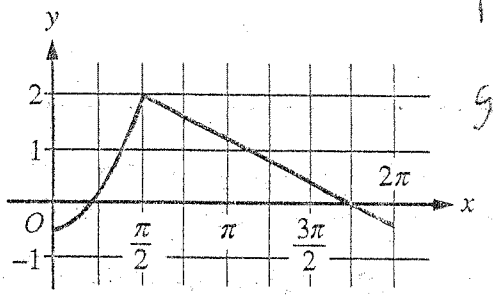
x	$f(x)$
0	1
$\pi/4$	$e^{\pi/4} \cdot \sqrt{2}/2$
$5\pi/4$	$e^{5\pi/4} \cdot (-\sqrt{2}/2)$
2π	$e^{2\pi}$

The minimum value is $f = -e^{5\pi/4} \cdot (\sqrt{2}/2)$.

After checking all endpoints and critical values as shown in the table on the left, $-e^{5\pi/4} \cdot \sqrt{2}/2$ was the only negative number, so it had to be the lowest on the whole graph in $0 \le x \le 2\pi$

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.

$f(x)$ is both continuous and differentiable at $x = \frac{\pi}{2}$.



Graph of g'

$f\left(\frac{\pi}{2}\right) = 0 = \lim_{x \to \frac{\pi}{2}} f(x)$

$g\left(\frac{\pi}{2}\right) = 0 = \lim_{x \to \frac{\pi}{2}} g(x)$

If $g'(x)$ is continuous across $\pi/2$, then $g(x)$ must be continuous there.

$$\lim_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)} = \lim_{x \to \frac{\pi}{2}} \frac{f'(x)}{g'(x)} = \frac{e^{\pi/2}\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right)}{2} = \frac{-e^{\pi/2}}{2}$$

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5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

$$= \frac{f(\pi) - f(0)}{\pi}$$

$f(\pi) = e^{\pi} \cdot \cos\pi$

$= -e^{\pi}$

$f(0) = e^{0} \cos 0$

$= 1$

$$= \frac{-e^{\pi} - 1}{\pi}$$

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

$f'(x) = e^x \cdot \cos x + (-\sin x) e^x$

$f'\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}} \cdot \cos\left(\frac{3\pi}{2}\right) - \sin\left(\frac{3\pi}{2}\right) \cdot e^{\frac{3\pi}{2}}$

$= e^{\frac{3\pi}{2}} \cdot 0 + (+1) e^{\frac{3\pi}{2}}$

$= e^{\frac{3\pi}{2}}$

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Continue question 5 on page 19.

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

$f'(x) = 0$

$0 = e^x \cos x - e^x \sin x$

$0 = e^x (\cos x - \sin x)$

$0 = \cos x - \sin x$

$\sin x = \cos x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$f(0) = 1$

$f(2\pi) = e^{2\pi}$

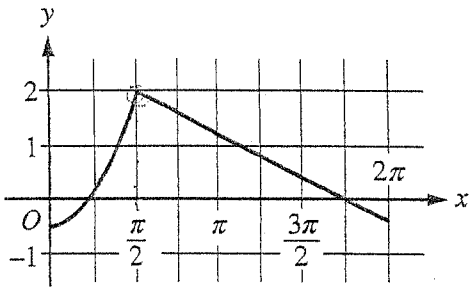
$f\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}} \cdot \frac{\sqrt{2}}{2}$

$f\left(\frac{5\pi}{4}\right) = e^{\frac{5\pi}{4}} \cdot -\frac{\sqrt{2}}{2}$

∴ the absolute minimum value of f on $0 \le x \le 2\pi$ is $-\frac{\sqrt{2}}{2} \cdot e^{\frac{5\pi}{4}}$

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.

$\lim_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)} = 0$



Graph of g'

$f'\left(\frac{\pi}{2}\right) =$

$e^{\frac{\pi}{2}} \cos\frac{\pi}{2} - e^{\frac{\pi}{2}} \sin\frac{\pi}{2}$

$-e^{\frac{\pi}{2}}$

L'Hopital's Rule

$\lim_{x \to \frac{\pi}{2}} \frac{f'(x)}{g'(x)} = \frac{-e^{\frac{\pi}{2}}}{2}$

$\lim_{x \to \frac{\pi}{2}} \frac{f'(x)}{g'(x)} = -\frac{1}{2} e^{\frac{\pi}{2}}$ because of L'Hopital's Rule

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5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

$$\text{av. } f = \frac{f(\pi) - f(0)}{\pi - 0}$$

$$= \frac{e^{\pi}\cos\pi - e^{0}\cos 0}{\pi}$$

$$= \frac{e^{\pi}(-1) - (1)(1)}{\pi}$$

$$= \frac{-e^{\pi} - 1}{\pi}$$

∴ average rate of change of f is $-\frac{e^{\pi}+1}{\pi}$

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

$$f'(x) = e^{x}\cos x - e^{x}\sin x = e^{x}(\cos x - \sin x)$$

$$f'\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}}\left(\cos\left(\frac{3\pi}{2}\right) - \sin\left(\frac{3\pi}{2}\right)\right)$$

$$= e^{\frac{3\pi}{2}}(0 - (-1))$$

$$= e^{3\pi/2}$$

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Continue question 5 on page 19.

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

$0 = e^x(\cos x - \sin x)$

$e^x = 0$

d.n.e.

$\cos x - \sin x = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}$

or $x = \frac{5\pi}{4}$

$f(0) = 1$

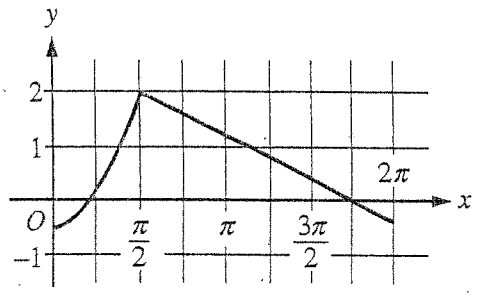
$f(2\pi) = e^{2\pi}$

$f\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}}\left(\frac{1}{\sqrt{2}}\right)$ } all positive

$f\left(\frac{5\pi}{4}\right) = e^{\frac{5\pi}{4}}\left(\frac{-1}{\sqrt{2}}\right)$ } negative

∴ There is an abs. min. at $x = \frac{5\pi}{4}$

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

$\lim_{x \to \pi/2} \frac{f'(x)}{g'(x)}$

$= \lim_{x \to \pi/2} \frac{e^x(\cos x - \sin x)}{g'(x)}$

$= \frac{e^{\pi/2}(0 - 1)}{2}$

$= \frac{-e^{\pi/2}}{2}$

$\therefore \lim_{x \to \pi/2} \frac{f(x)}{g(x)} = \frac{-e^{\pi/2}}{2}$

L'Hopital's Rule

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5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

$$\frac{1}{\pi - 0} \int_0^{\pi} e^x \cos x \, dx$$

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

$f = e^x \quad f' = e^x$

$g = \cos x \quad g' = -\sin x$

$f'(x) = e^x \cos(x) - e^x \sin(x)$

$f'\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}} \cos\left(\frac{3\pi}{2}\right) - e^{\frac{3\pi}{2}} \sin\left(\frac{3\pi}{2}\right)$

$f'\left(\frac{3\pi}{2}\right) = 0 + e^{\frac{3\pi}{2}} = \boxed{e^{\frac{3\pi}{2}}}$

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Continue question 5 on page 19.

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

$f(0) = 1$

$f(2\pi) = e^{2\pi}$

$f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} e^{\frac{\pi}{4}}$

$\boxed{f\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2} e^{\frac{5\pi}{4}}}$

$0 = e^x \cos(x) - e^x \sin(x)$

$0 = e^x(\cos(x) - \sin(x))$

$0 = \cos(x) - \sin(x)$

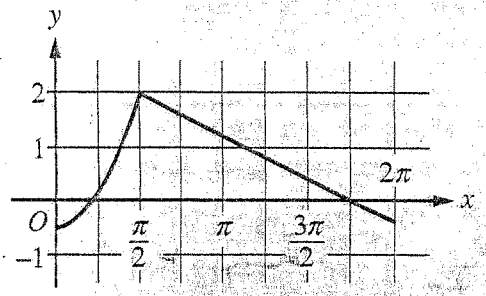
$\sin(x) = \cos(x)$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

The absolute minimum is at $\left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{2} e^{\frac{5\pi}{4}}\right)$ since it is the lowest y-value of all the critical points.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.

$\lim_{x \to \frac{\pi}{2}} \frac{e^x \cos x}{g(x)} = \frac{0}{0}$



Graph of g'

$\lim_{x \to \frac{\pi}{2}} e^x \cos x = 0$

$\lim_{x \to \frac{\pi}{2}} g(x) = 0$

By L'Hospital $\lim_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)} = \lim_{x \to \frac{\pi}{2}} \frac{f'(x)}{g'(x)}$

$\lim_{x \to \frac{\pi}{2}} \frac{e^x \cos x - e^x \sin x}{g'(x)} = \frac{0 - 1e^{\frac{\pi}{2}}}{2} = \boxed{\frac{e^{\frac{\pi}{2}}}{2}}$

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5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

$$\frac{f(\pi) - f(0)}{\pi - 0} = \frac{(e^{\pi}\cos\pi) - (e^{0}\cos 0)}{\pi} = \frac{(-e^{\pi}) - (1)}{\pi}$$

$$= \boxed{\frac{-e^{\pi} - 1}{\pi}}$$

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

$$f\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}}\cos\left(\frac{3\pi}{2}\right) = 0$$

$$f'(x) = e^x\cos x - e^x\sin x$$

$$f'\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}}\cos\left(\frac{3\pi}{2}\right) - e^{\frac{3\pi}{2}}\sin\left(\frac{3\pi}{2}\right) = 0 + e^{\frac{3\pi}{2}}$$

$$\boxed{m = e^{\frac{3\pi}{2}}}$$

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Continue question 5 on page 19.

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

$f'(x) = e^x \cos x - e^x \sin x = 0$

$e^x \cos x = e^x \sin x$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$f(0) = e^0 \cos(0) = 1$ $\quad f(\frac{\pi}{4}) = e^{\frac{\pi}{4}} \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} e^{\frac{\pi}{4}}$

$f(2\pi) = e^{2\pi} \cos(2\pi) = e^{2\pi}$ $\quad f(\frac{5\pi}{4}) = e^{\frac{5\pi}{4}} \cos(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2} e^{\frac{\pi}{4}}$

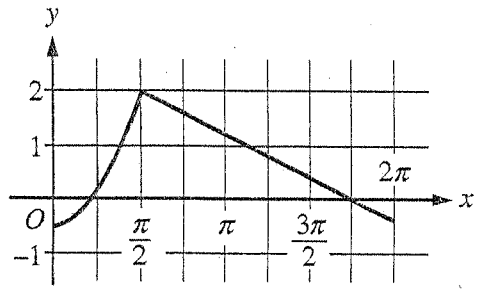
$f(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2} e^{\frac{\pi}{4}}$

x	y
0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} e^{\frac{\pi}{4}}$
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2} e^{\frac{\pi}{4}}$
2π	$e^{2\pi}$

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.

$\lim_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)}$

$\lim_{x \to \frac{\pi}{2}} \frac{f(\frac{\pi}{2})}{g(\frac{\pi}{2})} = \frac{0}{0}$



Graph of g'

$f(\frac{\pi}{2}) = e^{\frac{\pi}{2}} \cos(\frac{\pi}{2})$

$f(\frac{\pi}{2}) = 0$

$f'(\frac{\pi}{2}) = e^{\frac{\pi}{2}} \cos(\frac{\pi}{2}) - e^{\frac{\pi}{2}} \sin(\frac{\pi}{2})$

$f'(\frac{\pi}{2}) = -e^{\frac{\pi}{2}}$

L'Hop $\lim_{x \to \frac{\pi}{2}} \frac{(g(x))(f'(x)) - (f(x))(g'(x))}{(g(x))^2} = \lim_{x \to \frac{\pi}{2}} \frac{(g(\frac{\pi}{2}))(f'(\frac{\pi}{2})) - (f(\frac{\pi}{2}))(g'(\frac{\pi}{2}))}{(g(\frac{\pi}{2}))^2}$

$= \frac{(0)(-e^{\frac{\pi}{2}}) - (0)(2)}{(0)^2} = \frac{0}{0}$

$\lim_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)}$ DNE

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5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

where a → average rate of change

$$a = \frac{f(\pi) - f(0)}{\pi - 0}$$

$$a = \frac{(-e^{\pi}) - 1 \cdot 1}{\pi} = \frac{-e^{\pi}}{\pi}$$

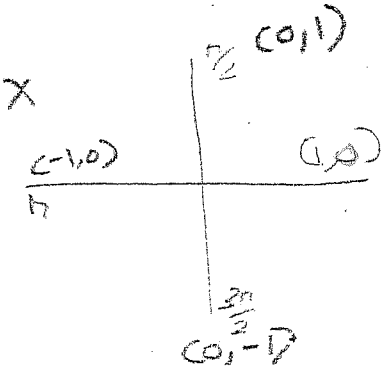
(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

$$f'(x) = e^x \cos x - e^x \sin x$$

$$f'(x) = e^x(\cos x - \sin x)$$

$$e^{\frac{3\pi}{2}}(0 - (-1))$$

$$= e^{\frac{3\pi}{2}}$$



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Continue question 5 on page 19.

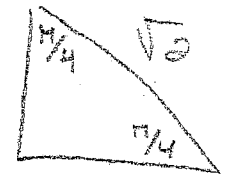
(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

$f'(x) = e^x \cos x - e^x \sin x$

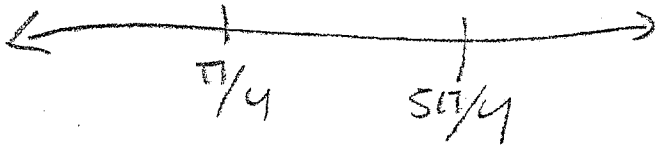
$0 = e^x(\cos x - \sin x)$

$e^x = 0$ $\quad$ $\cos x - \sin x = 0$

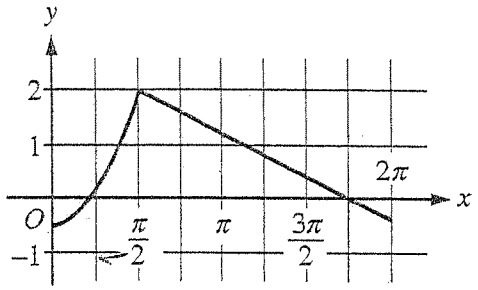
$x = \text{DNE}$ $\quad$ $x = \pi/4, \frac{5\pi}{4}$



S	A
T	C



(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

$\lim_{x \to \pi/2} \frac{f(x)}{g(x)} = \frac{\lim_{x \to \pi/2} f(x)}{\lim_{x \to \pi/2} g(x)} = \frac{e^x(0)}{0} = \frac{0}{0} = \text{DNE}$ ← since $g(x)$ is differentiable, it is continuous, so the $\lim_{x \to \pi/2} g(x) = g(\pi/2)$

b/c the answer is 0/0 the function may be able to be factored (the $\cos x$) may be able to be factored

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M

5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

$$\frac{dy}{dx} = \frac{f(\pi) - f(0)}{\pi - 0} \qquad \frac{\overset{(-1)}{[e^{\pi}\cos(\pi)]} - \overset{(1)}{[e^{0}\cos(0)]}}{\pi}$$

$$\frac{dy}{dx} = \frac{-e^{\pi} - 1}{\pi}$$

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

$f(x) = e^x \cos x \qquad u = e^x \quad v = \cos x$

$u' = e^x \quad v' = -\sin x$

$f'(x) = e^x \cos x + (-e^x \sin x) \qquad u'v + v'u$

$e^x \cos x + (-e^x \sin x)$

$f'\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}} \underset{(0)}{\cos\left(\frac{3\pi}{2}\right)} - e^{\frac{3\pi}{2}} \underset{(0)}{\sin\left(\frac{3\pi}{2}\right)}$

$f'\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}}(0 - 0)$

$f'\left(\frac{3\pi}{2}\right) = 0$

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Continue question 5 on page 19.

M

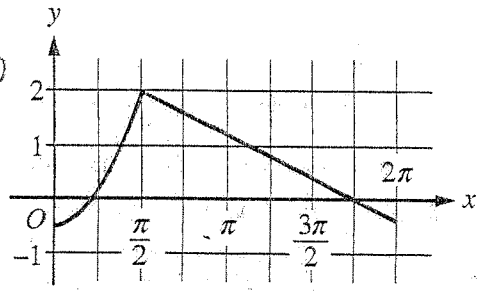
(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

$f(x) = e^x \cos x$

$f'(x) = e^x \cos x - e^x \sin x = 0$

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.

$\frac{f(\frac{\pi}{2})}{g(\frac{\pi}{2})} = \frac{0}{0}$ $\lim_{x \to \pi/2} = 0$



Graph of g'

$e^{\pi/2} \cos\left(\frac{\pi}{2}\right)$ 0

the $\lim_{x \to \pi/2} \frac{f(x)}{g(x)} = 0$

$\lim_{x \to \pi/2^-} \frac{f(x)}{g(x)} = \lim_{x \to \pi/2^+} \frac{f(x)}{g(x)}$

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5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

(-1,0) (1,0) (0,-1)

$e^0 \cos(0)$ $\quad e^\pi \cos\pi$

$(1)(1) = 1$ $\quad e^\pi(-1) = -e^\pi$

$(0, 1)$ $\quad (\pi, -e^\pi)$

$$\frac{1 + e^\pi}{0 - \pi} \rightarrow \boxed{\frac{-1 + e^\pi}{\pi}}$$

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

$(\frac{3\pi}{2}, 0)$

$$\boxed{y = -e^{3\pi/2}\left(x - \frac{3\pi}{2}\right)}$$

$f'(x) = e^x \cos x - e^x \sin x$

$f'(x) = e^x(\cos x - \sin x)$

$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}\left(\cos\left(\frac{3\pi}{2}\right) - \sin\left(\frac{3\pi}{2}\right)\right)$

$f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}(0 - 1)$

$f'\left(\frac{3\pi}{2}\right) = -e^{3\pi/2}$

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Continue question 5 on page 19.

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

$f'(x) = e^x(\cos x - \sin x)$

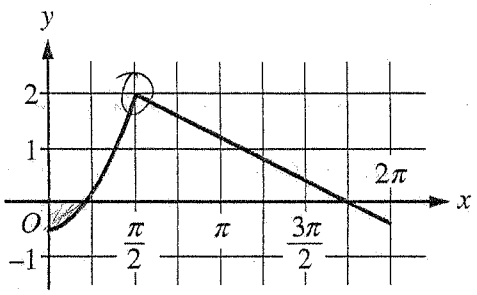
$0 = e^x(\cos x - \sin x)$

$e^x = 0$ doesn't exist

$\cos x - \sin x = 0$

$\cos x = \sin x$

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

$\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ does not exist because even if $\lim_{x \to \pi/2} f(x)$ had a value, the $\lim_{x \to \pi/2} g(x)$ does not exist.

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NO CALCULATOR ALLOWED

Q

5. Let f be the function defined by $f(x) = e^x \cos x$.

$e^0 = 1$

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

$$f(x) = e^x \cos x$$

$$\text{avg rate} = \frac{f(\pi) - f(0)}{\pi - 0} = \frac{(e^\pi \cos \pi) - (e^0 \cos 0)}{\pi - 0} = \frac{e^\pi \cos \pi - e^0 \cos 0}{\pi}$$

$$\frac{e^\pi(-1) - (1)(1)}{\pi} = \frac{-e^\pi - 1}{\pi}$$

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

$$f(x) = e^x \cos x$$

$$f\left(\frac{3\pi}{2}\right) = e^{\frac{3\pi}{2}} \cos\left(\frac{3\pi}{2}\right)$$

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Continue question 5 on page 19.

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

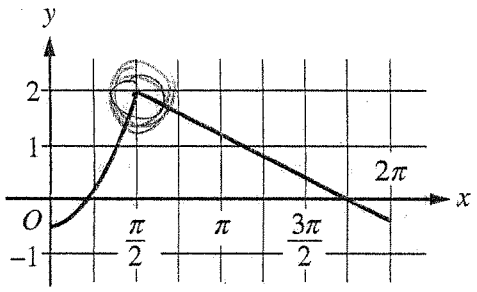
$f(x) = e^x \cos x$

$f'(x) = (e^x)(-\sin x) + (\cos x)(e^x)(1)$

$f'(x) = -e^x \sin x + e^x \cos x$

$e^x(-\sin x + \cos x)$

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.



Graph of g'

value $\lim_{x \to \frac{\pi}{2}} \frac{f(x)}{g(x)}$

Does not exist bc it is a corner

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AP® SAMPLE STUDENT RESPONSES AND SCORING NOTES
2018 AP CALCULUS AB

Question 5

Sample Identifier: N
Score: 9

- **The response earned 1-2-3-3.**
- In part (a) the response presented a difference quotient and earned the point for the answer $\frac{-e^{\pi}-1}{\pi}$.
- In part (b) the response earned the first point for the correct derivative of f, $f'(x) = e^x \cdot \cos x + e^x \cdot (-\sin x)$. The derivative point would have been earned by the first declaration of $f'(x)$ in the first line, but responses that go on to factor, must do so correctly. The derivative expression is factored correctly to $f'(x) = e^x(\cos x - \sin x)$, earning the point. The second point would have been earned for the slope $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cdot (0 + 1)$ in the second line, but responses that go on to simplify, must do so correctly. The slope expression is simplified correctly to $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$, earning the point.
- In part (c) the response earned the first point with $f'(x) = 0$. The second point is earned for identifying $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ as candidates in the first line. The third point is earned for the absolute minimum value $f = -e^{5\pi/4} \cdot \left(\sqrt{2}/2\right)$ and the global argument appealing to the behavior of f on the entire interval. The candidates test is used to justify the minimum value is the only negative extreme value on the interval.
- In part (d) the response earned the first point by stating g is continuous and showing the limits of f and g are equal to 0. The second point is earned for a limit attached to the ratio of derivatives, $\lim_{x \to \pi/2} \frac{f'(x)}{g'(x)}$. The third point would have been earned for the answer $\frac{e^{\pi/2}\left(\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\right)}{2}$ in the last line, but responses that go on to simplify, must do so correctly. The limit expression is simplified correctly to $\frac{-e^{\pi/2}}{2}$, earning the point.

Sample Identifier: L
Score: 8

- **The response earned 1-2-3-2.**
- In part (a) the response presented a difference quotient and earned the point for the answer $\frac{-e^{\pi}-1}{\pi}$.
- In part (b) the response earned the first point for the correct derivative of f, $f'(x) = e^x \cdot \cos x + -\sin x \cdot e^x$. The second point would have been earned for the slope $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cdot \cos\left(\frac{3\pi}{2}\right) - \sin\left(\frac{3\pi}{2}\right) \cdot e^{3\pi/2}$ in the second line, but responses that go on to simplify, must do so correctly. The slope expression is simplified correctly to $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$, earning the point.
- In part (c) the response earned the first point for setting the derivative equal to 0, $f'(x) = 0$. The second point is earned for identifying $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ as candidates in the sixth line on the left. The third point is earned for the absolute minimum value $-\frac{\sqrt{2}}{2} \cdot e^{5\pi/4}$ and the global argument appealing to the behavior of f on the entire interval by means of a candidates test.
- In part (d) the response did not earn the first point for omitting g is continuous and the limits of f and g are not shown to be equal to 0. The second point is earned for a limit attached to the ratio of derivatives, $\lim_{x \to \pi/2} \frac{f'(x)}{g'(x)}$. The third point would have been earned for the answer $\frac{-e^{\pi/2}}{2}$ in the third line,

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but responses that go on to simplify, must do so correctly. The limit is simplified correctly to $-\frac{1}{2}e^{\pi/2}$, earning the point.

Sample Identifier: E
Score: 7

- **The response earned 1-2-2-2.**
- In part (a) the response presented a difference quotient and would earn the point for the answer $\frac{e^{\pi}\cos\pi - e^{0}\cos 0}{\pi}$ in the second line, but responses that go on to simplify, must do so correctly. The average rate of change expression is simplified correctly to $\frac{-e^{\pi}-1}{\pi}$, earning the point.
- In part (b) the response would have earned the first point for the correct derivative of f, $f'(x) = e^{x}\cos x - e^{x}\sin x$. The second point would have been earned for the slope $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}\left(\cos\left(\frac{3\pi}{2}\right) - \sin\left(\frac{3\pi}{2}\right)\right)$ in the second line, but responses that go on to simplify, must do so correctly. The slope expression is simplified correctly to $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$, earning the point.
- In part (c) the response earned the first point for setting the derivative equal to 0, $0 = e^{x}(\cos x - \sin x)$. The second point is earned for identifying $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ as candidates. The third point is not earned for identifying the location of the absolute minimum value at $x = \frac{5\pi}{4}$ without identifying the minimum value. Although the response includes the minimum value in the work associated with the candidates test, it was necessary to identify the relevant information in that work, that the absolute minimum value is $e^{5\pi/4}\left(-\frac{1}{\sqrt{2}}\right)$, in order to earn the point.
- In part (d) the response does not verify the conditions that g is continuous and that $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} g(x) = 0$, so the response does not earn the point. The second point is earned on the first line for a limit attached to the ratio of derivatives, $\lim_{x\to\pi/2}\frac{f'(x)}{g'(x)}$. The third point would have been earned for the answer $\frac{e^{\pi/2}(0-1)}{2}$ in the third line, but responses that go on to simplify, must do so correctly. The limit is simplified correctly to $\frac{-e^{\pi/2}}{2}$, earning the point.

Sample Identifier: B
Score: 6

- **The response earned 0-2-3-1.**
- In part (a) the response presented an incorrect integral expression, $\frac{1}{\pi}\int_{0}^{\pi} e^{x}\cos x\,dx$ and does not earn the point.
- In part (b) the response earned the first point for the correct derivative of f, $f'(x) = e^{x}\cos x - e^{x}\sin x$. The second point would have been earned for the slope $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}\cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2}\sin\left(\frac{3\pi}{2}\right)$ in the second line, but responses that go on to simplify, must do so correctly. The slope expression is simplified correctly to $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}$, earning the point.
- In part (c) the response earned the first point for setting the derivative equal to 0, $0 = e^{x}\cos x - e^{x}\sin x$. The second point is earned for identifying $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ as candidates. The third point is earned for the absolute minimum value $f\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}e^{5\pi/4}$ and the global

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Question 5 (continued)

argument appealing to the behavior of the entire interval. The candidates test is used to determine that $-\frac{\sqrt{2}}{2}e^{5\pi/4}$ is the lowest y-value on the interval.

- In part (d) the response did not earn the first point for two reasons: omitting g is continuous and stating $\lim_{x\to\pi/2}\frac{f(x)}{g(x)}=\frac{0}{0}$. The second point is earned on the first line for a limit attached to the ratio of derivatives, $\lim_{x\to\pi/2}\frac{f'(x)}{g'(x)}$. The third point would have been earned for the expression $\frac{0-1e^{\pi/2}}{2}$, but responses that go on to simplify, must do so correctly. The limit is simplified incorrectly to $\frac{e^{\pi/2}}{2}$ and does not earn the point.

Sample Identifier: A
Score: 5

- **The response earned 1-2-2-0.**
- In part (a) the response presented a difference quotient and would have earned the point for the answer $\frac{(e^{\pi}\cos\pi)-(e^{0}\cos 0)}{\pi}$. The average rate of change point would have been earned by for this expression, but responses that go on to simplify, must do so correctly. The average rate of change expression is simplified correctly to $\frac{-e^{\pi}-1}{\pi}$, earning the point.
- In part (b) the response earned the first point for the correct derivative of f, $f'(x)=e^{x}\cos x-e^{x}\sin x$. The second point would have been earned for the slope $f'\left(\frac{3\pi}{2}\right)=e^{3\pi/2}\cos\left(\frac{3\pi}{2}\right)-e^{3\pi/2}\sin\left(\frac{3\pi}{2}\right)$ in the third line, but responses that go on to simplify, must do so correctly. The slope expression is simplified correctly to $m=e^{3\pi/2}$, earning the point.
- In part (c) the response earned the first point for setting the derivative equal to 0, $f'(x)=e^{x}\cos x-e^{x}\sin x=0$. The second point is earned for identifying $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$ as candidates. The third point is not earned because $-\frac{\sqrt{2}}{2}e^{\pi/4}$ is not the absolute minimum value of f on $0\le x\le 2\pi$.
- In part (d) the response did not earn the first point for two reasons: omitting g is continuous and stating $\lim_{x\to\pi/2}\frac{f(x)}{g(x)}=\frac{0}{0}$. The second and third point are not earned because the response does not apply L'Hospital's Rule.

Sample Identifier: J
Score: 4

- **The response earned 0-2-2-0.**
- In part (a) the response presented a difference quotient and would have earned the point for the expression $\frac{(-e^{\pi})-1\cdot 1}{\pi}$, but responses that go on to simplify, must do so correctly. The average rate of change expression is simplified incorrectly to $\frac{-e^{\pi}}{\pi}$ and does not earn the point.
- In part (b) the response earned the first point for the correct derivative of f, $f'(x)=e^{x}\cos x-e^{x}\sin x$. The derivative point would have been earned by the first declaration of $f'(x)$ in the first line, but responses that go on to factor, must do so correctly. The derivative expression is factored correctly to $f'(x)=e^{x}(\cos x-\sin x)$, earning the point. The second point would have been earned for the expression for slope $e^{3\pi/2}\big(0-(-1)\big)$ in the third line, but

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Question 5 (continued)

responses that go on to simplify, must do so correctly. The slope expression is simplified correctly to $e^{3\pi/2}$, earning the point.

- In part (c) the response earned the first point for setting the derivative equal to 0, $0 = e^x(\cos x - \sin x)$. The second point is earned for identifying $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ as candidates. The third point is not earned, because an absolute minimum value with a global argument is not presented.
- In part (d) the response did not earn the first point for two reasons: omitting g is continuous and stating $\lim_{x\to\pi/2} \frac{f(x)}{g(x)} = \frac{0}{0}$. The response confirmed that g is continuous but stated $\lim_{x\to\pi/2} \frac{f(x)}{g(x)} = \frac{0}{0}$ and does not earn the point. The second point is not earned without a limit attached to the ratio of derivatives, $\lim_{x\to\pi/2} \frac{f'(x)}{g'(x)}$. A maximum of one point can be earned, namely the first point, for responses with no limit notation attached to a ratio of derivatives. As a result of this error, the response was ineligible for the answer point.

Sample Identifier: M
Score: 3

- **The response earned 1-1-1-0.**
- In part (a) the response presented a difference quotient and earned the point for the answer $\frac{(e^\pi \cos \pi) - (e^0 \cos 0)}{\pi}$. The average rate of change point would have been earned by for this expression, but responses that go on to simplify, must do so correctly. The average rate of change expression is simplified correctly to $\frac{-e^\pi - 1}{\pi}$, earning the point.
- In part (b) the response earned the first point for the correct derivative of f, $f'(x) = e^x \cos x - e^x \sin x$. The second point would have been earned for the slope $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right)$ in the third line, but responses that go on to simplify, must do so correctly. The slope expression is simplified incorrectly to $f'\left(\frac{3\pi}{2}\right) = 0$ and does not earn the point.
- In part (c) the response earned the first point for setting the derivative equal to 0, $e^x \cos x - e^x \sin x = 0$. Because the response does not identify $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ as candidates, the second point is not earned. The third point is not earned, because an absolute minimum value with a global argument is not presented.
- In part (d) the response did not earn the first point for two reasons: omitting g is continuous and stating $\lim_{x\to\pi/2} \frac{f(x)}{g(x)} = \frac{0}{0}$. The second point is not earned without a limit attached to the ratio of derivatives, $\lim_{x\to\pi/2} \frac{f'(x)}{g'(x)}$. A maximum of one point can be earned, namely the first point, for responses with no limit notation attached to a ratio of derivatives. As a result of this error, the response was ineligible for the answer point.

Sample Identifier: P
Score: 2

- **The response earned 0-1-1-0.**
- In part (a) the response does not earn the point because $\frac{-1+e^\pi}{\pi}$ is an incorrect answer.

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Question 5 (continued)

- In part (b) the response earned the first point for the correct derivative of f, $f'(x) = e^x \cos x - e^x \sin x$. The derivative point would have been earned by the first declaration of $f'(x)$ in the first line, but responses that go on to factor, must do so correctly. The derivative expression is factored correctly to $f'(x) = e^x(\cos x - \sin x)$, earning the point. The second point would have been earned for the slope $f'\left(\frac{3\pi}{2}\right) = e^{3\pi/2}\left(\cos\left(\frac{3\pi}{2}\right) - \sin\left(\frac{3\pi}{2}\right)\right)$ in the third line, but responses that go on to simplify, must do so correctly. The slope expression is simplified incorrectly to $f'\left(\frac{3\pi}{2}\right) = -e^{3\pi/2}$ and does not earn the point.
- In part (c) the response earned the first point for setting the derivative equal to 0, $0 = e^x(\cos x - \sin x)$. The response does not identify $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ as candidates and does not earn the second point. The third point is not earned, because an absolute minimum value with a global argument is not presented.
- In part (d) the response incorrectly states that the limit does not exist and does not present work that earns any points.

Sample Identifier: Q
Score: 1

- **The response earned 1-0-0-0.**
- In part (a) the response presented a difference quotient and would have earned the point for the answer $\frac{(e^{\pi}\cos\pi) - (e^0\cos 0)}{\pi}$. The average rate of change point would have been earned by for this expression, but responses that go on to simplify, must do so correctly. The average rate of change expression is simplified correctly to $\frac{-e^{\pi}-1}{\pi}$, earning the point.
- In part (b) the response earned no points for not identifying the correct derivative of f. As a result of this error, the response was ineligible for the slope point.
- In part (c) the response does not earn the first point by not setting the derivative equal to 0. The second point is not earned for not identifying $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ as candidates. The third point is not earned, because an absolute minimum value with a global argument is not presented.
- In part (d) the response incorrectly states that the limit does not exist and does not present work that earns any points.

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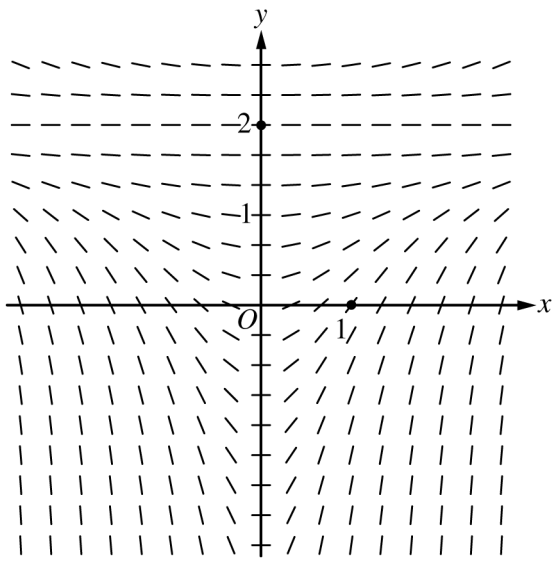
6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

(a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

STOP

END OF EXAM

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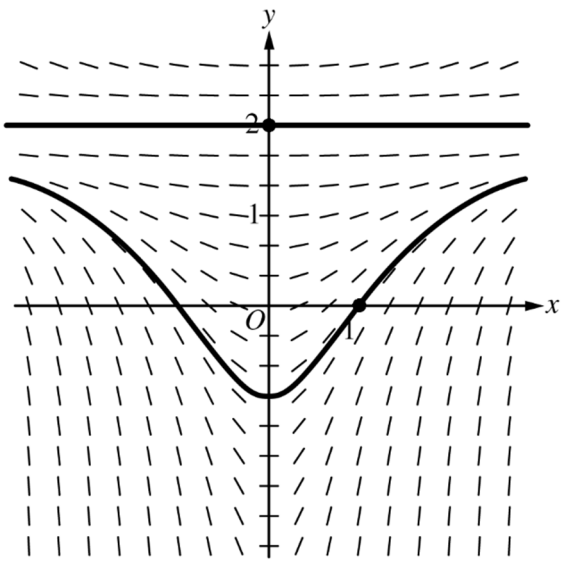
Question 6

(a)



$2 : \begin{cases} 1 : \text{solution curve through } (0, 2) \\ 1 : \text{solution curve through } (1, 0) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = \dfrac{4}{3}$

An equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ is $y = \dfrac{4}{3}(x - 1)$.

$f(0.7) \approx \dfrac{4}{3}(0.7 - 1) = -0.4$

$2 : \begin{cases} 1 : \text{equation of tangent line} \\ 1 : \text{approximation} \end{cases}$

(c) $\dfrac{dy}{dx} = \dfrac{1}{3}x(y-2)^2$

$\displaystyle\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x\,dx$

$\dfrac{-1}{y-2} = \dfrac{1}{6}x^2 + C$

$\dfrac{1}{2} = \dfrac{1}{6} + C \Rightarrow C = \dfrac{1}{3}$

$\dfrac{-1}{y-2} = \dfrac{1}{6}x^2 + \dfrac{1}{3} = \dfrac{x^2+2}{6}$

$y = 2 - \dfrac{6}{x^2+2}$

Note: this solution is valid for $-\infty < x < \infty$.

$5 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration and uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: $0/5$ if no separation of variables

Note: max $3/5$ [1-2-0-0] if no constant of integration

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TTT

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

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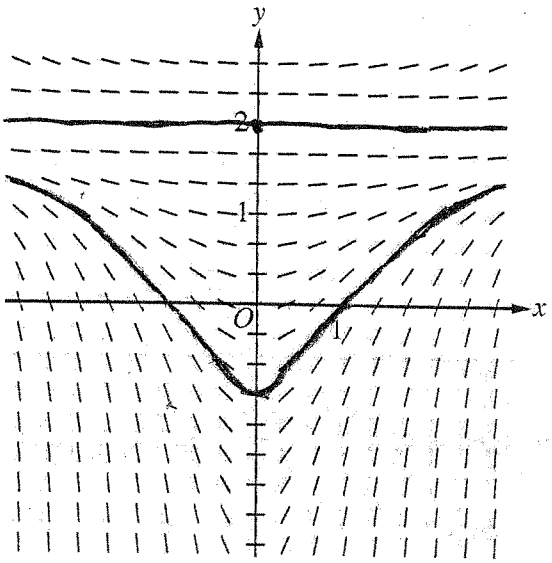

(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

$y - 0 = \frac{4}{3}(x-1)$

$y = \frac{4}{3}(x-1)$ line of tangent at $x=1$

point: $(1, 0)$

slope: $= \frac{1}{3}(1)(0-2)^2$

$= \frac{4}{3}$

$y(0.7) = \frac{4}{3}(0.7 - 1)$

$= \frac{4}{3}(-\frac{3}{10})$

$= -\frac{4}{10} = -\frac{2}{5} = y(0.7)$ approximation

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Continue question 6 on page 21.

TTT

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

at $f(1)=0$

$$\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$$

$$dy = \frac{1}{3}x(y-2)^2\,dx$$

$$(y-2)^{-2}\,dy = \frac{1}{3}x\,dx$$

$$-(y-2)^{-1} = \frac{1}{6}x^2 + C$$

$$\frac{1}{-(0-2)} = \frac{1}{6}(1)^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C$$

$$\frac{1}{3} = C$$

$$\frac{1}{-(y-2)} = \frac{x^2}{6} + \frac{1}{3}$$

$$\frac{1}{-y+2} = \frac{x^2+2}{6}$$

$$\rightarrow 6 = (-y+2)(x^2+2)$$

$$\frac{6}{x^2+2} = -y+2$$

$$\frac{6}{x^2+2} - 2 = -y$$

$$2 - \frac{6}{x^2+2} = y$$

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CCC

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

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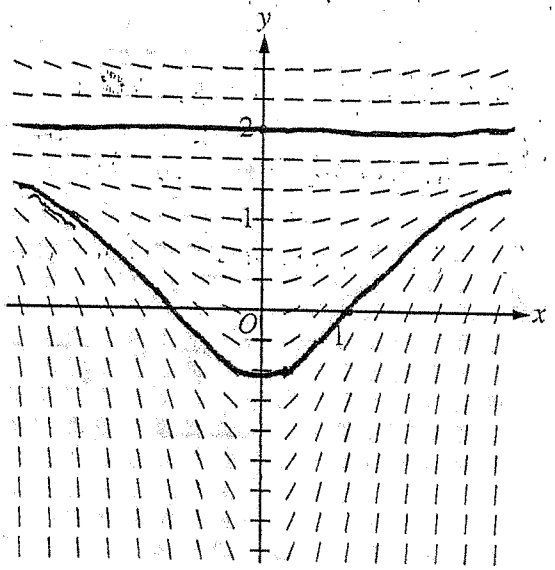

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$\therefore \frac{dy}{dx} = \frac{1}{3}x(y-2)^2$

$\int \frac{1}{(y-2)^2}\,dy = \int \frac{1}{3}x\,dx$

$\frac{-1}{y-2} = \frac{1}{6}x^2 + C$

$\frac{-1}{0-2} = \frac{1}{6}(1)^2 + C$

$\frac{1}{3} = C$

$\therefore \frac{dy}{dx} = \frac{1}{3}x(y-2)^2$

$= \frac{1}{3}(1)(0-2)^2$

$= \frac{1}{3}(1)(-2)^2$

$= \frac{4}{3}$

$\therefore y - y_1 = m(x - x_1)$

$y = \frac{4}{3}(x-1)$

$\therefore y = \frac{4}{3}(0.7-1)$

$= \frac{4}{3}(-0.3)$

$= \frac{-1.2}{3} = \frac{-12}{30}$

$= -6/15$

$\therefore y = \frac{4}{3}(x-1)$

$y(0.7) \simeq -6/15$

1 0.3 ~ 4 1.2

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Continue question 6 on page 21.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$$\therefore \frac{dy}{dx} = \frac{1}{3}x(y-2)^2$$

$$dy = \frac{1}{3}x(y-2)^2 \cdot dx$$

$$\frac{dy}{(y-2)^2} = \frac{1}{3}x\,dx$$

$$\int \frac{1}{(y-2)^2}\,dy = \int \frac{1}{3}x\,dx$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + C$$

* Let $u = (y-2)$, $du = dy$

$u^{-2} \Rightarrow -1u^{-1}$

Conditions

$$\therefore \frac{-1}{0-2} = \frac{1}{6}(1)^2 + C$$

$$\frac{-1}{-2} = \frac{1}{6} + C$$

$$\frac{1}{2} = \frac{1}{6} + C$$

$$\frac{1}{2} - \frac{1}{6} = C$$

$$\frac{3}{6} - \frac{1}{6} = C$$

$$\frac{1}{3} = C$$

$$\therefore \frac{-1}{y-2} = \frac{1}{6}x^2 + \frac{1}{3}$$

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WWW

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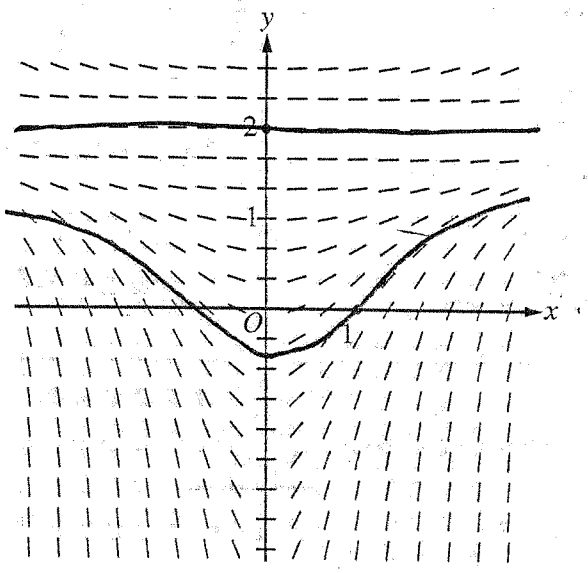

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$\frac{dy}{dx} = \frac{1}{3}(1)(0-2)^2$

$\frac{1}{3} \cdot 1 \cdot 4 = \frac{4}{3} \frac{dy}{dx}$

$y - 0 = \frac{4}{3}(x-1)$

$y = \frac{4}{3}x - \frac{4}{3}$

$.7 \cdot 4 = 3\overline{)2.8}$ → $6.9\overline{3}$, 27, 10, 9

$.9\overline{3} - 1.\overline{3}$

1.333 − $.933$ = $.400$

$f(0.7) \approx 0.400$

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Continue question 6 on page 21.

WWW

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$$\frac{1}{(y-2)^2}\,dy = \frac{1}{3}x\,dx$$

$$(y-2)^{-2} = \frac{1}{3}x\,dx$$

$$\downarrow$$

$$-\frac{1}{(y-2)} = \frac{x^2}{6} + C$$

$$-\frac{1}{(-2)} = \frac{1}{6} + C$$

$$\frac{1}{2} - \frac{1}{6} = C$$

$$\frac{1}{3} = C$$

$$-\frac{1}{(y-2)} = \frac{x^2}{6} + \frac{1}{3}$$

$$-(y-2) = 6x^{-2} + 3$$

$$y - 2 = -6x^{-2} - 3$$

$$\boxed{y = -6x^{-2} - 1}$$

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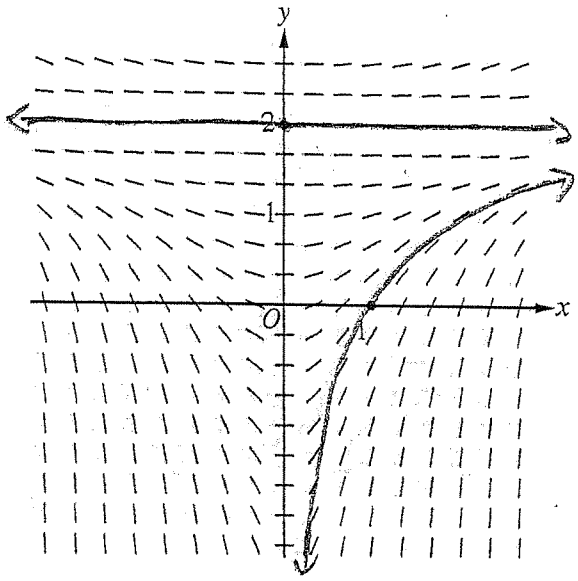

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$\frac{dy}{dx} = \frac{1}{3}(-2)^2 = \frac{1}{3}(4) = \frac{4}{3}$

$y - 0 = \frac{4}{3}(x-1)$, $y = \frac{4}{3}(x-1)$

$f(0.7) \approx f(1) + f'(1)(1-0.7)$

$\approx f(1) + f'(1)$

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Continue question 6 on page 21.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$$\int \frac{1}{(y-2)^2}\,dy = \int \frac{1}{3}x\,dx, \qquad \int u^{-2}\,du = \frac{1}{3}\int x\,dx$$

$u = y-2$
$du = dy$

$$\frac{u^{-1}}{-1} = \frac{1}{3}\left(\frac{x^2}{2}\right)+C, \qquad \frac{-1}{y-2} = \frac{x^2}{6}+C$$

$$\frac{1}{2} = \frac{1}{6}+C$$

$$C = \frac{1}{2}-\frac{1}{6} = \frac{3}{6}-\frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{-1}{y-2} = \frac{x^2+2}{6}, \qquad (y-2)(x^2+2) = -6$$

$$y-2 = \frac{-6}{x^2+2}$$

$$y = \frac{-6}{x^2+2} - 2$$

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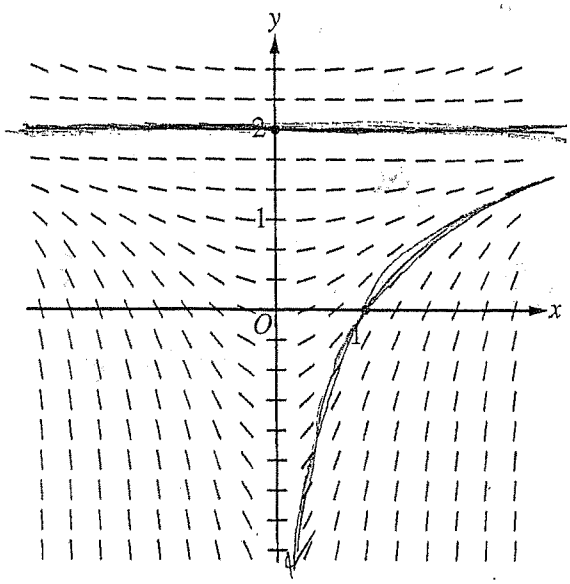

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$y = f(x)$

$\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$ separate variables

$\frac{dy}{(y-2)^2} = \frac{\frac{1}{3}x(y-2)^2\,dx}{(y-2)^2}$

$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x\,dx$

$u = y-2$ $\int u^{-2}$ u^{-1}

$du = dx$

$\int \frac{1}{u^2} = \int u^{-2}$ $-(y-2)^{-1} = \int u^{-2}$

$\frac{1}{-(y-2)} = \frac{1}{2-y}$

$\frac{1}{2-y} = \frac{x^2}{6} + C$

Plug in initial condition $(1, 0)$ to find C

$\frac{1}{2} = \frac{1}{6} + C$

$-\frac{1}{6} \quad -\frac{1}{6}$

$C = \frac{2}{6} = \frac{1}{3}$

$\frac{1}{2-y} = \frac{x^2}{6} + \frac{1}{3}$

$2-y = \frac{6}{x^2} + 3$

$\boxed{y = -\frac{6}{x^2} - 1}$

approximate $f(.7)$

$\boxed{y \approx \frac{-6}{(.7)^2} - 1}$

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Continue question 6 on page 21.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

(1, 0)

$\frac{dy}{dx} = \frac{1}{3}x(y-2)^2(dx)$

$\frac{dy}{(y-2)^2} = \frac{\frac{1}{3}x(y-2)^2 dx}{(y-2)^2}$ separated variables

$\frac{dy}{(y-2)^2} = \frac{1}{3}x\,dx$

$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x\,dx$

$u = y-2$ $\quad \frac{x^2}{6} + C$

$du = dx$

$\int u^{-2} = -u^{-1}$

$-(y-2)^{-1} = \frac{1}{2-y}$

$\frac{1}{2-y} = \frac{x^2}{6} + C$

Plug in initial condition (1, 0)

$\frac{1}{2} = \frac{1}{6} + C$

$-\frac{1}{6} \quad -\frac{1}{6}$

$\frac{3}{6} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3} = C$

$\frac{1}{2-y} = \frac{x^2}{6} + \frac{1}{3}$

$2 - y = \frac{6}{x^2} + 3$

$-y = \frac{6}{x^2} + 1$

$\boxed{y = -\frac{6}{x^2} - 1}$

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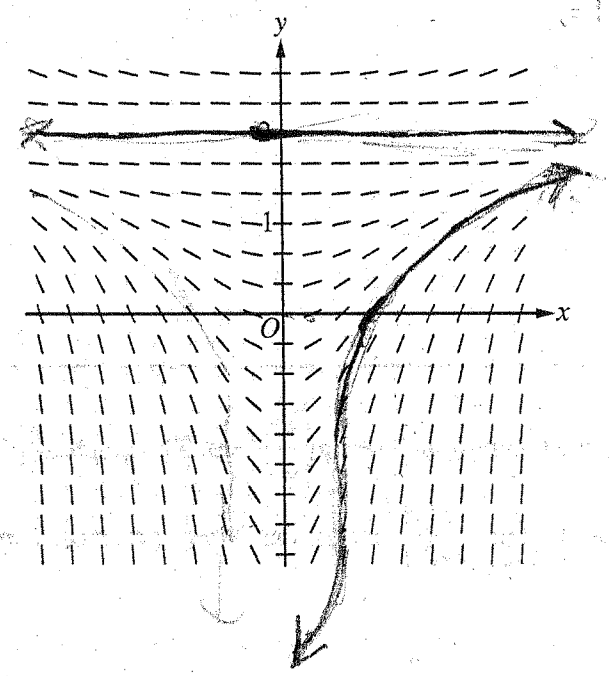

(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

$\frac{dy}{dx} = \frac{1}{3}(1)(0-2)^2$

$= \frac{1}{3}(4)$

$m = \frac{4}{3}$

$y_1 - y_2 = m(x_1 - x_2)$

$y - 0 = m(x-1)$ ← slope

$y - 0 = \frac{4}{3}(x-1)$

$f(0.7) \approx \frac{4}{3}(.7-1)$

$y \approx \frac{4}{3}\left(-\frac{3}{10}\right)$

$y \approx -\frac{12}{30}$ OR $-\frac{2}{5}$

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Continue question 6 on page 21.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$$\frac{dy}{dx} = \frac{1}{3} \times (y-2)^2$$

$$\frac{dy}{(y-2)^2} = \frac{1}{3} \times dx$$

$$\frac{1}{2(y-2)} = \frac{1}{3} + C$$

$$\frac{1}{2(0)-4} = \frac{1}{3} C$$

$$\frac{1}{-4} = \frac{1}{3} + C$$

$$-\frac{7}{12} = C$$

$$\frac{1}{2y-4} = \frac{1}{3} - \frac{7}{12}$$

$$\frac{1}{2y-4} = \frac{-11}{4}$$

$$4 = -2y + 4$$

$$0 = -2y$$

$$\boxed{y = 0}$$

$$-\frac{3}{12} - \frac{4}{12}$$

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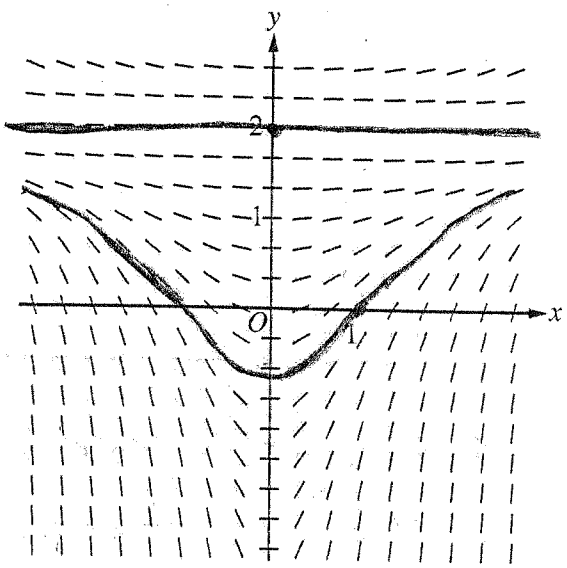

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point = (1, 0)

slope = $\frac{1}{3}(x)(y-2)^2$

$\frac{1}{3}(1)(0-2)^2$

$\frac{1}{3}(-2)^2 \rightarrow \frac{1}{3}4 = \frac{4}{3}$

tangent line equation = $y - 0 = \frac{4}{3}(x-1)$

$f(.7) = \frac{4}{3}(.7-1)$

$= \frac{4}{3}(-.3)$

$f(.7) \approx -.225$

$.75 \times -.3 = -.225$

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222

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$\frac{1}{3} \times (y-2)^2$

$\frac{1}{3} 1(0-2)^2$

$\frac{1}{3}(-2)^2$

$\frac{1}{3} \cdot 4 = \frac{4}{3}$?

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HHHH

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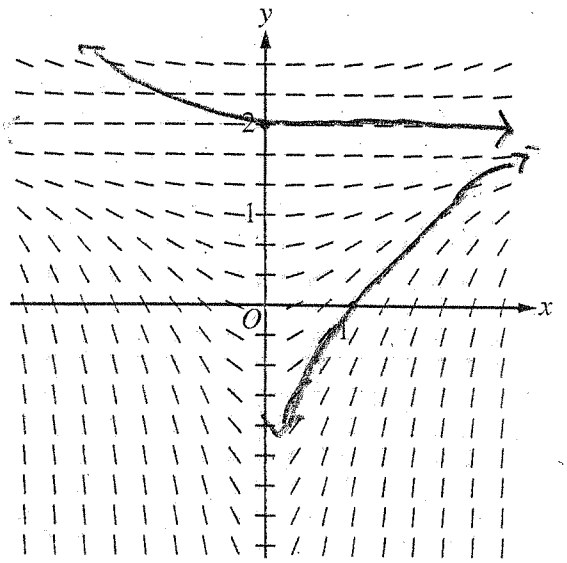

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(1) $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$

$\frac{1}{3}(1)(0-2)^2$

$\frac{1}{3}(4) = \frac{4}{3}$ (slope)

(1,0)

0.7
4
2.8

(2) $y = mx + b$

$0 = \frac{4}{3}(1) + b$

$0 = \frac{4}{3} + b$

$b = -\frac{4}{3}$

2.8
−4.0
−2.8

(3) $y = \frac{4}{3}x - \frac{4}{3}$

$y = \frac{4}{3}(0.7) - \frac{4}{3}$

$y = \frac{2.8}{3} - \frac{4}{3}$

$f(0.7) = \frac{-2.8}{3}$

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Continue question 6 on page 21.

AHHH

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

(1, 0)

$$\frac{dy}{dx} = \frac{1}{3} \times (y-2)^2$$

$$\frac{dy}{(y-2)^2} = \frac{1}{3} x \, dx$$

$$\int \frac{1}{(y-2)^2} dy = \int \frac{1}{3} x \, dx$$

$$\frac{1}{y-2} = \frac{1}{3} + C$$

$$\frac{1}{0-2} = \frac{1}{3} + C$$

$$-\frac{1}{2} = \frac{1}{3} + C$$

$$-\frac{1}{3}$$

$$C = -\frac{5}{6}$$

$y = mx + b$

$$0 = m(1) - \frac{5}{6}$$

$$\frac{5}{6} = m$$

$$y = \frac{5}{6} x - \frac{5}{6}$$

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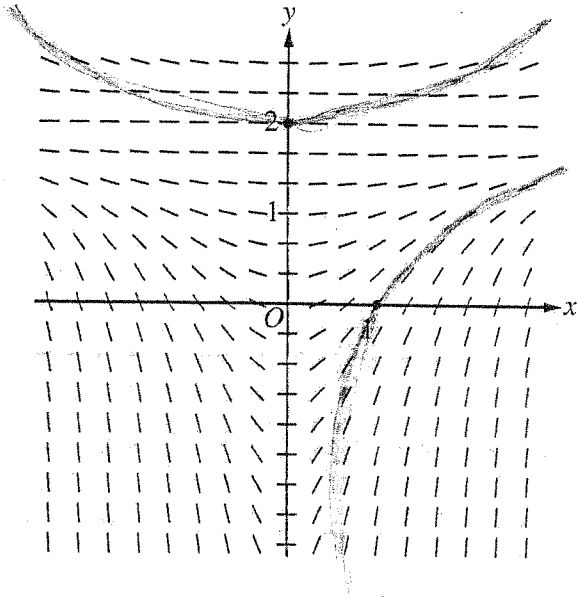
6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$.

(a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.

$x = 1$
$y = 0$
$m = \frac{4}{3}$

$\frac{1}{3}(1)(0-2)^2$
$\frac{1}{3}(-2^2)$
$\frac{1}{3}(4)$
$\frac{4}{3}$

$y - 0 = \frac{4}{3}(x-1)$

$f(0.7) \approx 0.4$

$y = \frac{4}{3}(0.7-1)$
$y = \frac{4}{3}(-.3)$

$\begin{array}{r} 4 \\ \times .3 \\ \hline 1.2 \\ \hline 3 \end{array}$

y

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Continue question 6 on page 21.

(c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

$\int F(x)\,dx$

$\int x(y-2)^2$

$\frac{x(y-2)^2}{3}$

$\int \frac{x(y-2)^3}{3} \cdot \frac{1}{3}$

$\frac{\frac{3}{1} \cdot \frac{1}{3}}{}$

$\frac{x(y-2)^3}{9}$

$\frac{1}{3}(1)(0-2)^2$

$\frac{1}{3}(-2)^2$

$\frac{1}{3}(4)$

$\boxed{\frac{4}{3}}$

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Question 6

Sample Identifier: TTT
Score: 9

- **The response earned 2-2-5.**
- In part (a) the response earned the first point with the solution curve y=2 correctly drawn through the point (0, 2). The response earned the second point with the solution curve through (1, 0) that follows the slope lines and extends to both boundaries of the slope field.
- In part (b) the response earned the first point with the appearance of equation of the tangent line $y - 0 = \frac{4}{3}(x - 1)$. The response would have earned the second point for the approximation of $f(0.7)$ by stating $y(0.7) = \frac{4}{3}(0.7 - 1)$. The response continues and correctly simplifies the approximation to $-\frac{2}{5}$ and earned the second point.
- In part (c) the response earned the first point with the correct separation of variables in the equation, $(y-2)^{-2}dy = \frac{1}{3}x\,dx$ in line 3 on the left. The response earned the second and third points with the correct antiderivatives, $-(y-2)^{-1} = \frac{1}{6}x^2 + C$ in line 4 on the left. The response earned the fourth point by both including C with the antiderivatives on line 4 and substituting 1 for x and 0 for y in line 5. The response earned the fifth point by presenting a correct solution, $y = 2 - \frac{6}{x^2+2}$.

Sample Identifier: CCC
Score: 8

- **The response earned 2-2-4.**
- In part (a) the response earned the first point with the solution curve y=2 correctly drawn through the point (0, 2). The response earned the second point with the solution curve through (1, 0) that follows the slope lines and extends to both boundaries of the slope field.
- In part (b) the response earned the first point with the appearance of equation of the tangent line $y = \frac{4}{3}(x - 1)$. The response earned the second point for the approximation of $f(0.7)$ by presenting $y(0.7) \approx -\frac{6}{15}$. The response has additional work, some related to part (c) and some related to part (b) that neither earned nor prohibited the earning of points.
- In part (c) the response earned the first point with the correct separation of variables in the equation, $\frac{dy}{(y-2)^2} = \frac{1}{3}x\,dx$ in line 3 on left. The response earned the second and third points with the correct antiderivatives, $-\frac{1}{y-2} = \frac{1}{6}x^2 + C$ in line 5 on the left. The response earned the fourth point with both the constant of the integration in line 5 on the left and substituting 1 for x and 0 for y in the line under the word "Conditions." The response did not earn the fifth point without solving for $y = f(x)$. The response contains some crossed out work that indicates it should not be read in part (c).

Sample Identifier: WWW
Score: 7

- **The response earned 2-1-4.**
- In part (a) the response earned the first point with the solution curve y=2 correctly drawn through the point (0, 2). The response earned the second point with the solution curve through (1, 0) that follows the slope lines and extends to both boundaries of the slope field.

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Question 6 (continued)

- In part (b) the response would have earned the first point with $y - 0 = \frac{4}{3}(x - 1)$ in the first line. The correct simplification $y = \frac{4}{3}x - \frac{4}{3}$ earned the first point. The response does not earn the second point because it reported the approximation $f(0.7)$ as 0.400, which is the additive inverse of the correct linear approximation.
- In part (c) the response earned the first point with the correct separation of variables with the equation $\frac{1}{(y-2)^2}dy = \frac{1}{3}x\,dx$. The response earned the second and third points with the correct antiderivatives, $-\frac{1}{y-2} = \frac{x^2}{6} + C$, in line 3 on the left. The response earned the fourth point with the constant of integration in line 3 and substituting 1 for x and 0 for y in line 4 on the left. The response did not earn the fifth point because it did not correctly solve for $y = f(x)$.

Sample Identifier: II
Score: 6

- **The response earned 1-1-4.**
- In part (a) the response earned the first point with the solution curve y=2 correctly drawn through the point (0, 2). The response did not earn the second point since the curve through (1,0) does not follow the slope lines correctly from the left boundary to the right boundary of the slope field.
- In part (b) the response would have earned the first point with $y - 0 = \frac{4}{3}(x - 1)$ in the first line. The correct simplification $y = \frac{4}{3}(x - 1)$ earned the first point. The response does not earn the second point by reporting the approximation $f(0.7)$ symbolically. If the response had continued and substituted in the correct values for $f(1)$ and $f'(1)$, the second point would have been earned.
- In part (c) the response earned the first point with the correct separation of variables in the equation $\int \frac{1}{(y-2)^2}dy = \int \frac{1}{3}x\ dx$. The response earned the second and third points with the correct antiderivatives, $\frac{-1}{y-2} = \frac{x^2}{6} + C$, at the end of line 2. The response earned the fourth point with both the constant of integration in line 2 and substituting 1 for x and 0 for y resulting in $\frac{1}{2} = \frac{1}{6} + C$ in line 3. The response mishandled the 2 while solving and does not earn the fifth point with an incorrect final solution for $y = f(x)$.

Sample Identifier: Z
Score: 5

- **The response earned 1-0-4.**
- In part (a) the response earned the first point with the solution curve y=2 correctly drawn through the point (0, 2). The response did not earn the second point since the curve through (1,0) does not follow the slope lines correctly from the left boundary to the right boundary of the slope field.
- In part (b) the response did not earn the first point since there is never a tangent line equation for $f(x)$ presented. The response reports an incorrect value for the approximation of $f(0.7)$ by evaluating an equation that is not a tangent line for $f(x)$ at $x = 0.7$.
- In part (c) the response earned the first point with the correct separation of variables in the equation, $\frac{dy}{(y-2)^2} = \frac{1}{3}x\,dx$ in line 3. The response earned the second and third points with the correct antiderivatives, $\frac{1}{2-y} = \frac{x^2}{6} + C$ in line 9. The response earned the fourth point with both the constant of integration in line 9 and substituting 1 for x and 0 for y resulting in $\frac{1}{2} = \frac{1}{6} + C$ in line 11. The response does not earn the fifth point with an incorrect final solution for $y = f(x)$.

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Question 6 (continued)

Sample Identifier: P
Score: 4

- **The response earned 1-2-1.**
- In part (a) the response earned the first point with the solution curve $y = 2$ correctly drawn through the point $(0, 2)$. The response did not earn the second point since the curve through $(1,0)$ does not follow the slope lines correctly from the left boundary to the right boundary of the slope field.
- In part (b) the response earned the first point with the appearance of equation of the tangent line $y - 0 = \frac{4}{3}(x - 1)$. The response earned the second point with the approximation of $f(0.7)$ by evaluating the tangent line at $x = 0.7$. The response would have earned the second point with the presentation of $\frac{4}{3}(.7 - 1)$, continued to simplify correctly and earned the point with the presentation of $\frac{-2}{5}$.
- In part (c) the response earned the first point with the correct separation of variables in the equation, $\frac{dy}{(y-2)^2} = \frac{1}{3}x\,dx$ in line 2. The response has an incorrect constant ½ instead of -1 in the attempted antiderivative for the expression of y. The response does not have the correct antiderivative of $\frac{1}{3}x$. The response has mistakes in both attempted antiderivatives and the response did not earn the second or third point. The response has not earned either antiderivative point and is not eligible for the fourth or fifth point.

Sample Identifier: ZZZ
Score: 3

- **The response earned 2-1-0.**
- In part (a) the response earned the first point with the solution curve $y = 2$ correctly drawn through the point $(0, 2)$. The response earned the second point with the solution curve through $(1, 0)$ that follows the slope lines and extends to both boundaries of the slope field.
- In part (b) the response earned the first point with the presentation of $y - 0 = \frac{4}{3}(x - 1)$. The response reports an incorrect value for the approximation of $f(0.7)$ and the second point was not earned. Had the response not incorrectly simplified ${}^{4}/_{3}\,(.7 - 1)$ or ${}^{4}/_{3}\,(-.3)$ as $f(.7) \approx -.225$ the response would have earned the second point.
- In part (c) the response does not separate the variables. With no separation of variables, the response is not eligible for any points in part (c).

Sample Identifier: HHHH
Score: 2

- **The response earned 0-1-1.**
- In part (a) the response does not have the correct curve, $y = 2$, through $(0, 2)$ and did not earn the first point. The response has a curve through $(1, 0)$ that does not follow the slope lines and does not extend from the left boundary to the right boundary of the slope field. The response has not earned the second point.
- In part (b) the response earned the first point with the presentation of $y = \frac{4}{3}x - \frac{4}{3}$ on the top line on the right. The response reports an incorrect value for the approximation of $f(0.7)$ and the second point was not earned. Had the response not incorrectly simplified $\frac{4}{3}(0.7) - \frac{4}{3}$ or $\frac{2.8}{3} - \frac{4}{3}$ as $-\frac{2.8}{3}$ the response would have earned the second point.

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Question 6 (continued)

- In part (c) the response earned the first point with the correct separation of variables in the equation $\frac{dy}{(y-2)^2} = \frac{1}{3}x\,dx$ in line 2. The response has an incorrect constant 1 instead of -1 in the attempted antiderivative for the expression of y. The response does not have the correct antiderivative of $\frac{1}{3}x$. The response has mistakes in both attempted antiderivatives and the response did not earn the second or third point. The response has not earned either antiderivative point and is not eligible for the fourth or fifth point.

Sample Identifier: AAAA
Score: 1

- **The response earned 0-1-0.**
- In part (a) the response does not have the correct curve, $y = 2$, through $(0, 2)$ and did not earn the first point. The response has a curve through $(1, 0)$ that does not follow the slope lines and does not extend from the left boundary to the right boundary of the slope field. The response has not earned the second point.
- In part (b) the response earned the first point with the presentation of $y - 0 = \frac{4}{3}(x - 1)$. The second point was not earned because the response reports the additive inverse value for the approximation, $f(0.7) \approx 0.4$, and the second point was not earned.
- In part (c) the response does not separate the variables. With no separation of variables, the response is not eligible for any points in part (c).

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CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

1. People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \le t \le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

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2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

   (a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

   (b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

   (c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

   (d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

END OF PART A OF SECTION II

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Question 2

(a) $p'(25) = -1.179$

At a depth of 25 meters, the density of plankton cells is changing at a rate of -1.179 million cells per cubic meter per meter.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

(b) $\int_0^{30} 3p(h)\,dh = 1675.414936$

There are 1675 million plankton cells in the column of water between $h = 0$ and $h = 30$ meters.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_{30}^{K} 3f(h)\,dh$ represents the number of plankton cells, in millions, in the column of water from a depth of 30 meters to a depth of K meters.

The number of plankton cells, in millions, in the entire column of water is given by $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh$.

Because $0 \le f(h) \le u(h)$ for all $h \ge 30$,

$3\int_{30}^{K} f(h)\,dh \le 3\int_{30}^{K} u(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 3 \cdot 105 = 315.$

The total number of plankton cells in the column of water is bounded by $1675.415 + 315 = 1990.415 \le 2000$ million.

$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{compares improper integral} \\ 1 : \text{explanation} \end{cases}$

(d) $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 757.455862$

The total distance traveled by the boat over the time interval $0 \le t \le 1$ is 757.456 (or 757.455) meters.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{total distance} \end{cases}$

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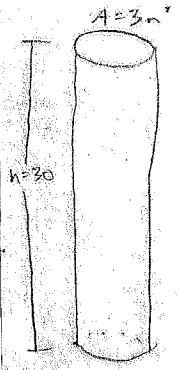
2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given. $p(h)$: m. cells/m^3

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'(25) = -1.179$ million cells/meter^3/m

At 25 meters below the surface of the water, the density of plankton cells is decreasing at a rate of 1.179 million cells/meter^3/m

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?



$\int_0^{30} A\,p(h)\,dh$

$3\int_0^{30} p(h)\,dh$

$= 1675.414$ million cells

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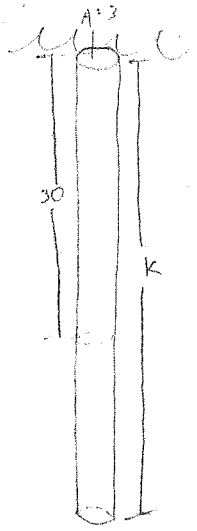
Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.



$$C(h) = \int_0^{30} A\,p(h)\,dh + \int_{30}^{K} A\,f(h)\,dh$$

$$C(h) = 1675.414 + 3\int_{30}^{K} f(h)\,dh$$

If $K = \infty$, then $3\int_{30}^{\infty} f(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 315$ million cells

Therefore, the maximum value of $3\int_{30}^{K} f(h)\,dh = 315$.

$C_{max} = 1675.414 + 315 = 1990.414$ million cells

∴ The maximum possible number of cells in a column infinitely deep would be 1990.414 million cells < 2000 million cells.

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$$D = \int_0^1 \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt = \int_0^1 \sqrt{[662\sin(5t)]^2 + [880\cos(6t)]^2}\,dt$$

$= 757.455$ meters

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2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'(25) = -1.179$ millions of cells per cubic meter per meter.

at depth $h = 25$ meters, the amount of plankton cells in the sea is decreasing at a rate of 1.179 millions of cells per cubic meter per meter.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$3\int_0^{30} p(h)\,dh = 3(558.47165) = 1675.4114$

1675 million plankton cells

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Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$3\int_0^{30} p(h)\,dh + 3\int_{30}^{K} f(h)\,dh = 1675 + 3\int_{30}^{K} f(h)\,dh$

Because $f(h) \le u(h)$ for $h > 30$,

$\int_{30}^{K} f(h)\,dh \le \int_{30}^{K} u(h)\,dh < \int_{30}^{\infty} u(h)\,dh = 105$

Therefore, $\int_{30}^{K} f(h)\,dh < 105$ and

$3\int_{30}^{K} f(h)\,dh < 315$

and by adding the value of $3\int_0^{30} p(h)\,dh$ to each side, the result is

$1675 + 3\int_{30}^{K} f(h)\,dh < 1990$

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$\int_0^1 \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt = 757.455$ meters

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GO ON TO THE NEXT PAGE.

C
1 of 2

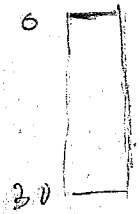
2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'(25) \approx -1.179$ millions of cells per cubic meter per meter

At a depth of 25 m, the density of plankton cells is decreasing at a rate of 1.179 million cells per cubic meter per meter

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?



$$\int_0^{30} 3p(h)\,dh = 3\int_0^{30} \left(.2h^2 e^{-.0025h^2}\right)dh \approx$$

1675 million plankton cells

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Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$$3\int_0^{30} p(h)\,dh + 3\int_{30}^{\infty} f(h)\,dh$$

Because $f(h)$ is nonnegative and less than or equal to $u(h)$, its definite integral is also less than or equal to that of $u(h)$. So, the max possible value of $\int_{30}^{\infty} f(h)\,dh$ is 105. If you plug in this value, along with the value you got found in part b,

$$1675 + 3(105) = 1990 < 2000$$

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$$\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt \approx 370235.588 \text{ m}$$

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2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p(h) = 0.2h^2e^{-0.0025h^2}$

$p'(h) = (0.4x - 0.001x^3)(0.997503)^{x^2}$

$p'(25) = -1.179$

At a depth of 25 meters, the density of plankton cells is decreasing at a rate of 1.179 million cells per cubic meter per meter.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$P = 3\int_0^{30} 0.2h^2e^{-0.0025h^2}\,dh = 1675.415$

$A = 3$

1675 million plankton cells

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Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$$p(h) = 3\int_0^{30} 0.2h^2 e^{-0.0025h^2}\,dh + 3\int_{30}^{\infty} u(h)\,dh$$

$$p(h) = 1675 + 3 \cdot 105 \le 1990 < 2000$$

Since the total number of plankton above 30 meters deep is has about 1675 million plankton cells and below 30 meters deep, there are about 315 million plankton cells, in total, this is less than 2000 million

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$$\int_0^1 \sqrt{(662\sin(5t))^2 + (880\cos(6t))^2} = 757.456$$

757.456 meters

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E

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'(25) = -1.179$

$p'(25)$ represents the rate at which the density of plankton cells in millions is decreasing at 1.179 millions of cells per cubic meter^2

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$3\int_0^{30} p(h)\,dh$

$= 1675.414$ cells

≈ 1675 cells (in millions)

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Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$$\#\text{ of plankton cells} = \int_0^{30} f(h)\,dh + \int_{30}^{K} u(h)\,dh$$

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(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$$\text{total distance} = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

$$= 757.455 \text{ meters}$$

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GO ON TO THE NEXT PAGE.

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p(h) = 0.2h^2e^{-.0025h^2}$

$p'(25) = \frac{d}{dh} p(h) \Big|_{h=25} = \boxed{-1.179 \text{ mil. cells/cubic meter}^2}$

$p'(25)$ is the rate at which the density of plankton is changing when $h = 25$

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$\int_0^{30} .2h^2e^{-.0025h^2} dh = 558.472$

$\frac{\text{million cells}}{m^3} = 558.472 \text{ cells per } m^3 \times 90 \text{ m}^3 = \boxed{50262.48 \text{ million cells}}$

$V = 30 \cdot 3 = 90$

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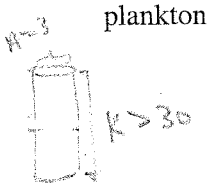
Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.



$90\int_0^{30}(.2h^2e^{-.005h^2})dh + 3K\int_{30}^{K} u(h)dh$

$V_1 = 3K$

$V_2 = 90$

$V = 3K + 90$

the # of cells is less than 2000 million because $\int_{30}^{\infty} u(h)dh = 105$ and $u(h) \ge f(h)$ and $3(105) < 2000$ million.

$3K\int_{30}^{K} f(h)dh$

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

Total Distance $= \int_0^1 \sqrt{(662\sin(5t))^2 + (880\cos(6t))^2}\,dt = 751.456$ m

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(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$p'(25) = -1.179$ millions of cells per cubic meter^2

The rate at which density in millions of plankton cells per cubic meter is decreasing at $p'(25)$ (25 meters deep) by -1.179 millions of cells per cubic meter^2

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$A = \pi r^2$

$\int_0^{30} [3]\, dh$ $\quad 3x \Big|_0^{30}$

$= 90$ million plankton cells

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Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$\int_0^{30}[3]\,dh + \int_{30}^{\infty}[u(h)]\,dh$

$90 + 105 = 195 < 2000$

195 million $<$ 2000 million

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(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$\int_0^1 \left[\sqrt{(662\sin(5t))^2 + (880\cos(6t))^2}\right] dt$

$= 757.456$ meters

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(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

$\frac{d}{dh} 0.2h^2e^{-0.0025h^2} = -1.179$ cells/m^3/m^3

At a depth of 25 meters, the density of the plankton cells is decreasing 1.179 millions cells per cubic meter per cubic meter.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

$3\left(\frac{1}{2}\int_0^{30}\left(0.2h^2e^{-0.0025h^2}\right)^2\right) =$

$1.5(13399.470) = 20099$ million plankton cells

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Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$3\left(\frac{1}{2}\int_0^{30}\left(0.2h^2 e^{-0.0025h^2}\right)^2 dh\right) + K\int_{30}^{\infty} u(h)\,dh$

2009.9 million $+ K(105$ million$)$.

The number of plankton cells are less than or equal to 2000 million because the density of the plankton cells is decreasing during the height.

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$\int_0^1 \sqrt{(662\sin 5t)^2 + (880\cos 6t)^2}$

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2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

graph $p(h)$ on calculator, then use $\frac{dy}{dx}$ button

$p'(h) = -1.179$ $\frac{\text{density}}{\text{meter}}$

This means at a depth of 25 meters, the density of the plankton is decreasing at a rate of 1.179.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

area of cross section = 3

$\int_0^{30} 3\,dt = 90$ million

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Continue question 2 on page 7.

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\,dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

$\int_{30}^{\infty} 3p(h)$

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

$\frac{dy}{dt} = \frac{880\cos 6t}{662\sin 5t}$ $\int_0^1 \left|\frac{880\cos 6t}{662\sin 5t}\right| dt =$

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AP® SAMPLE STUDENT RESPONSES AND SCORING NOTES
2018 AP CALCULUS BC

Question 2

Sample Identifier: A
Score: 9

- **The response earned 2-2-3-2.**
- In part (a) the response earned the first point for correctly presenting -1.179. The second point was earned by addressing the value of 25 and identifying that the density was decreasing at a rate of 1.179 with correct units.
- In part (b) the response earned the first point by presenting a definite integral with the correct integrand. The second point was earned by computing 1675.414 and presenting it as their final answer. The response was not penalized for presenting more decimal information than requested.
- In part (c) the response earned the first point by presenting and boxing the correct integral expression, using the numeric value of 1675 for the first integral. The response then correctly compared $\int_{30}^{\infty} f(h)dh$ to $\int_{30}^{\infty} u(h)dh$ to earn the second point. The response then computed and correctly identified the upper bound as 1990.414 to earn the third point.
- In part (d) the response earned the first point by presenting a correct integral setup for the distance traveled. The response presented the correct answer to three decimal places and earned the second point.

Sample Identifier: B
Score: 8

- **The response earned 1-2-3-2.**
- In part (a) the response earned the first point by presenting -1.179. The response did not earn the second point because it addressed the amount of plankton rather than the density of plankton.
- In part (b) the response earned the first point by presenting a definite integral with the correct integrand. The second point was earned by presenting 1675.
- In part (c) the response earned the first point by presenting the correct integral expression. The response then correctly compared $\int_{30}^{K} f(h)dh$ to $\int_{30}^{\infty} u(h)dh$ to earn the second point. The response then computed and correctly identified the upper bound as 1990 and earned the third point.
- In part (d) the response earned the first point by presenting a correct integral setup for the distance traveled. The response presented the correct answer to three decimal places and earned the second point.

Sample Identifier: C
Score: 7

- **The response earned 2-2-2-1.**
- In part (a) the response earned the first point for correctly presenting -1.179. The second point was earned by addressing the value of 25 and identifying that the density was decreasing at a rate of 1.179 with correct units.
- In part (b) the response earned the first point by presenting a definite integral with the correct integrand. The second point was earned by presenting 1675.
- In part (c) the response did not earn the first point because the second integral does not have an upper limit of K. The response then correctly compared the maximum value of $\int_{30}^{\infty} f(h)dh$ to 105 and earned the second point. The response did compute the value of 1990 and did earn the third point.

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Question 2 (continued)

- In part (d) the response earned the first point by presenting a correct integral setup for the distance traveled. The response presented an incorrect answer and did not earn the second point.

Sample Identifier: D
Score: 6

- **The response earned 2-2-0-2.**
- In part (a) the response earned the first point by presenting -1.179. The expression the response provides for $p'(h)$ is the CAS version provided by some graphing calculators. The second point was earned by addressing the value of 25 and identifying that the density was decreasing at a rate of 1.179 with correct units.
- In part (b) the response earned the first point by presenting a definite integral with the correct integrand. The second point was earned by presenting 1675.
- In part (c) the response never addressed $f(h)$ and was not eligible for either of the first two points. The response incorrectly identified 1990 as the number of plankton rather than the upper bound on the number of plankton and did not earn the third point.
- In part (d) the response earned the first point by presenting a correct integral setup for the distance traveled. The response presented the correct answer and earned the second point.

Sample Identifier: E
Score: 5

- **The response earned 1-2-0-2.**
- In part (a) the response earned the first point by presenting -1.179. The response did not earn the second point because it never addressed the meaning of the value of 25 as well as having incorrect units.
- In part (b) the response earned the first point by presenting a definite integral with the correct integrand. The second point was earned by presenting 1675.
- In part (c) the response presented an incorrect integral expression and did not earn the first point. No additional work was presented and the response did not earn either of the last two points.
- In part (d) the response earned the first point by presenting a correct integral setup for the distance traveled. The response presented the correct answer and earned the second point.

Sample Identifier: F
Score: 4

- **The response earned 1-1-0-2.**
- In part (a) the response earned the first point by presenting -1.179. The response did not earn the second point because the units were incorrect.
- In part (b) the response earned the first point by presenting a correct integral setup. The response improperly addressed the constant and did not earn the second point.
- In part (c) the response presented an incorrect integral expression and did not earn the first point. The response never compares an integral of $f(h)$ to $\int_{30}^{\infty} u(h)dh$ and did not earn the second point. An incorrect numeric value was obtained and never identified as the bound so the response did not earn the last point.
- In part (d) the response earned the first point by presenting a correct integral setup for the distance traveled. The response presented the correct answer and earned the second point.

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Question 2 (continued)

Sample Identifier: G
Score: 3

- **The response earned 1-0-0-2.**
- In part (a) the response earned the first point by presenting -1.179. The response did not earn the second point because the units were incorrect and the response combined decreasing with a negative answer.
- In part (b) the response did not earn the first point because the integrand was incorrect. The incorrect integrand was not eligible for the answer point, so the response did not earn the second point.
- In part (c) the response imported the incorrect integrand from part b, but the integrand was not eligible to be considered in part c, also the second integral did not have an upper bound of K, was missing the three, and the integrand was incorrect so the first point was not earned. The response never made an improper integral comparison between $f(h)$ and $u(h)$ and did not earn the second point. The numeric bound never addressed the missing three and did not earn the last point.
- In part (d) the response earned the first point by presenting a correct integral setup for the distance traveled. The response presented the correct answer and earned the second point.

Sample Identifier: H
Score: 2

- **The response earned 1-0-0-1.**
- In part (a) the response earned the first point by presenting -1.179. The response did not earn the second point because the units were incorrect and the rate was not specified.
- In part (b) the response presented a definite integral with an incorrect integrand and did not earn either point.
- In part (c) the response presented an incorrect integral expression and did not earn the first point. There was never a comparison of an integral of $f(h)$ to the improper integral of $u(h)$ so the second point was not earned. A numeric upper bound less than 2000 was never produced so the response did not earn the third point.
- In part (d) the response earned the first point by presenting a correct integral setup for the distance traveled. The response is not penalized for the missing dt. The response never presented a numeric answer and did not earn the second point.

Sample Identifier: I
Score: 1

- **The response earned 1-0-0-0.**
- In part (a) the response earned the first point by presenting -1.179. The response did not earn the second point because the units were never specified.
- In part (b) the response presented a definite integral with an incorrect integrand and did not earn either point.
- In part (c) the response presented an incorrect integral expression and did not earn the first point. No additional work was presented and the response did not earn either of the last two points.
- In part (d) the response set up an integral with an incorrect integrand and did not earn the first point. A numeric answer was not provided and the response did not earn the second point.

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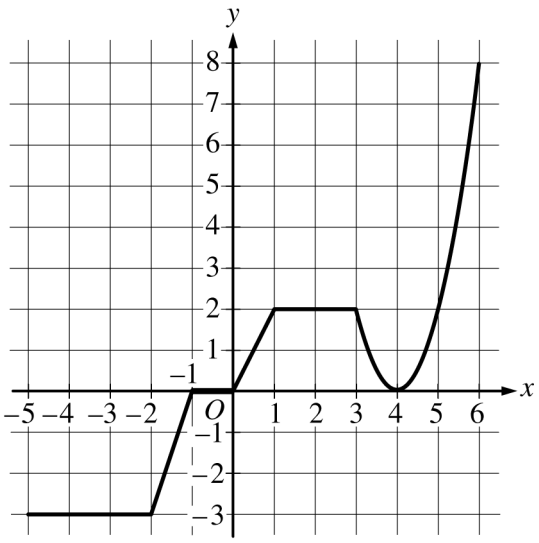
CALCULUS BC

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

   (a) If $f(1) = 3$, what is the value of $f(-5)$?

   (b) Evaluate $\int_1^6 g(x)\,dx$.

   (c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

   (d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

4. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

   (a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

   (b) Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

   (c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the average height of the tree over the time interval $2 \le t \le 10$.

   (d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \dfrac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

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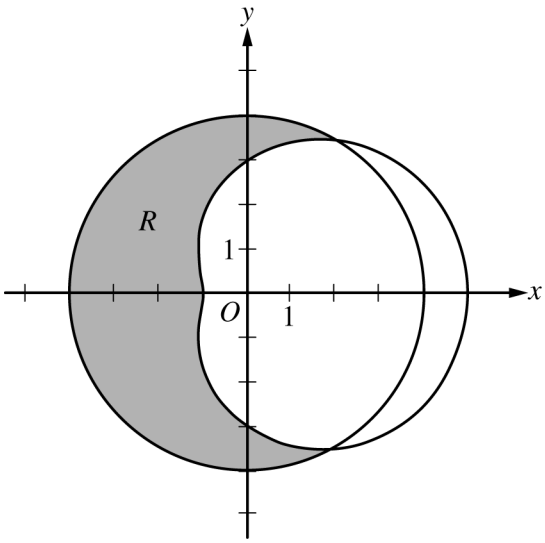

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

   (a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

   (b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

   (c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

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Question 5

(a) $\text{Area} = \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left(4^2 - (3 + 2\cos\theta)^2\right)d\theta$

$3 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \end{cases}$

(b) $\frac{dr}{d\theta} = -2\sin\theta \Rightarrow \left.\frac{dr}{d\theta}\right|_{\theta=\pi/2} = -2$

$r\left(\frac{\pi}{2}\right) = 3 + 2\cos\left(\frac{\pi}{2}\right) = 3$

$y = r\sin\theta \Rightarrow \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$

$x = r\cos\theta \Rightarrow \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$

$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/2} = \frac{-2\sin\left(\frac{\pi}{2}\right) + 3\cos\left(\frac{\pi}{2}\right)}{-2\cos\left(\frac{\pi}{2}\right) - 3\sin\left(\frac{\pi}{2}\right)} = \frac{2}{3}$

The slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$ is $\frac{2}{3}$.

— OR —

$y = r\sin\theta = (3 + 2\cos\theta)\sin\theta \Rightarrow \frac{dy}{d\theta} = 3\cos\theta + 2\cos^2\theta - 2\sin^2\theta$

$x = r\cos\theta = (3 + 2\cos\theta)\cos\theta \Rightarrow \frac{dx}{d\theta} = -3\sin\theta - 4\sin\theta\cos\theta$

$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/2} = \frac{3\cos\left(\frac{\pi}{2}\right) + 2\cos^2\left(\frac{\pi}{2}\right) - 2\sin^2\left(\frac{\pi}{2}\right)}{-3\sin\left(\frac{\pi}{2}\right) - 4\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)} = \frac{2}{3}$

The slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$ is $\frac{2}{3}$.

$3 : \begin{cases} 1 : \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta \\ \quad \text{or } \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta \\ 1 : \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} = -2\sin\theta\cdot\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{dr}{dt}\cdot\frac{1}{-2\sin\theta}$

$\left.\frac{d\theta}{dt}\right|_{\theta=\pi/3} = 3\cdot\frac{1}{-2\sin\left(\frac{\pi}{3}\right)} = \frac{3}{-\sqrt{3}} = -\sqrt{3}$ radians per second

$3 : \begin{cases} 1 : \frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt} \\ 1 : \frac{d\theta}{dt} = \frac{dr}{dt}\cdot\frac{1}{-2\sin\theta} \\ 1 : \text{answer with units} \end{cases}$

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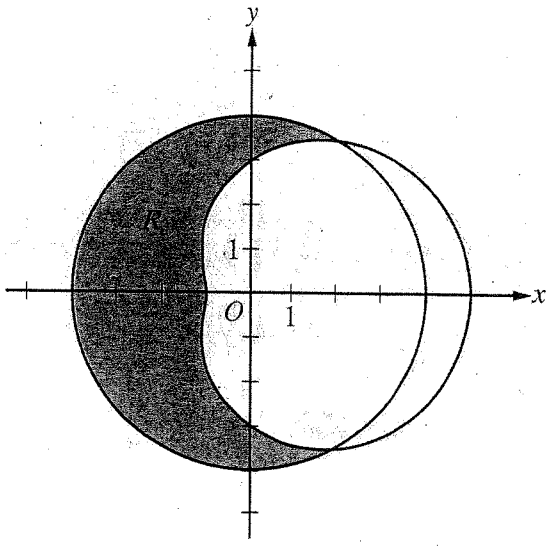

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$R = \frac{1}{2}\int_{\pi/3}^{5\pi/3}\left(16 - (3+2\cos\theta)^2\right)d\theta$$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$x = (3+2\cos\theta)\cos\theta$

$x = 3\cos\theta + 2\cos^2\theta$

$\frac{dx}{d\theta} = -3\sin\theta + 4\cos\theta(-\sin\theta)$

$= 3\sin\frac{\pi}{2} - 4\cos\frac{\pi}{2}\sin\frac{\pi}{2}$

$= -3$

$y = (3+2\cos\theta)\sin\theta$

$y = 3\sin\theta + 2\cos\theta\sin\theta$

$\frac{dy}{d\theta} = 3\cos\theta + 2\cos^2\theta - 2\sin^2\theta$

$\frac{dy}{d\theta} = 3\cos\frac{\pi}{2} + 2\cos^2\left(\frac{\pi}{2}\right) - 2\sin^2\left(\frac{\pi}{2}\right)$

$= -2$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2}{-3} = \frac{2}{3}$

The slope is $\frac{2}{3}$ at $\theta = \frac{\pi}{2}$.

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$r = 3 + 2\cos\theta$

$\frac{dr}{dt} = -2\sin\theta\frac{d\theta}{dt}$, $\frac{dr}{dt} = 3$ $\theta = \frac{\pi}{3}$

$3 = -2\sin\frac{\pi}{3}\left(\frac{d\theta}{dt}\right)$

$\frac{d\theta}{dt} = \frac{-3}{2} \cdot \frac{1}{\sqrt{3}/2}$

$= \frac{-3}{2} \cdot \frac{2}{\sqrt{3}}$

$\frac{d\theta}{dt} = \frac{-3}{\sqrt{3}}$ radians/second $= \frac{-3\sqrt{3}}{3} = -\sqrt{3}$ radians/second

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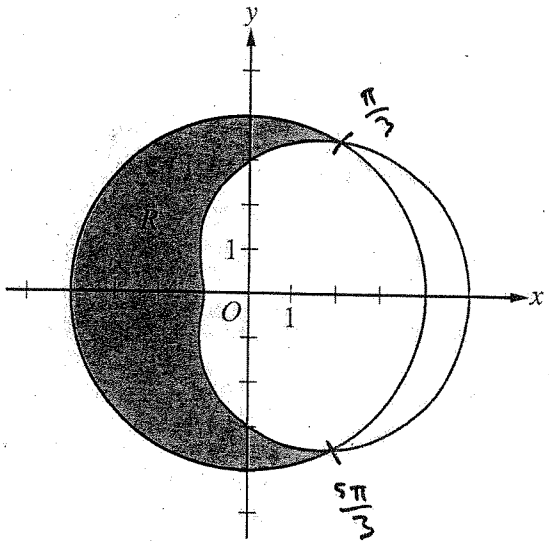

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$R = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \left((4^2) - (3 + 2\cos\theta)^2 \right) d\theta$$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$x = r\cos\theta$

$y = r\sin\theta$

$\frac{x}{\cos\theta} = 3 + 2\cos\theta$

$x = 3\cos\theta + 2\cos^2\theta$

$\frac{dx}{d\theta} = -3\sin\theta + 4\cos\theta(-\sin\theta)$

$\frac{y}{\sin\theta} = 3 + 2\cos\theta$

$y = 3\sin\theta + 2\sin\theta\cos\theta$

$\frac{dy}{d\theta} = 3\cos\theta + 2\cos^2\theta - 2\sin^2\theta$

$\frac{dy}{dx} = \frac{3\cos\theta + 2\cos^2\theta - 2\sin^2\theta}{-3\sin\theta - 4\cos\theta\sin\theta}$

$\frac{dy}{dx} = \frac{3(0) + 2(0) - 2(1)}{-3(1) - 4(0)(1)}$

$= \frac{-2}{-3}$

slope $= \frac{2}{3}$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$r = 3 + 2\cos\theta$

$\frac{dr}{dt} = -2\sin\theta\,\frac{d\theta}{dt}$

$3 = -2\sin\left(\frac{\pi}{3}\right)\frac{d\theta}{dt}$

$3 = -2\left(\frac{\sqrt{3}}{2}\right)\frac{d\theta}{dt}$

$3 = -\sqrt{3}\,\frac{d\theta}{dt}$

$\frac{d\theta}{dt} = -\frac{3}{\sqrt{3}}$

$\frac{d\theta}{dt} = -\frac{\sqrt{3}\cdot\sqrt{3}}{\sqrt{3}}$

$\frac{d\theta}{dt} = -\sqrt{3}$ units per second.

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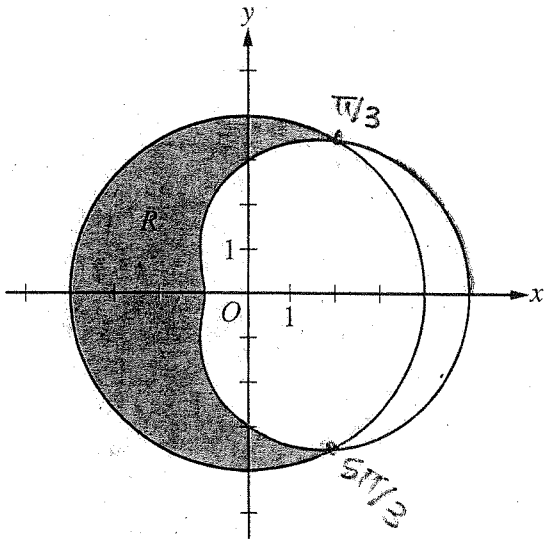

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$\int_{\pi/3}^{5\pi/3} \left(\frac{1}{2}(4)^2\right) - \left(\frac{1}{2}(3+2\cos\theta)^2\right) d\theta$$

area of polar $= \int_a^b \frac{1}{2} r^2 \, d\theta$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$x = r\cos\theta = (3 + 2\cos\theta)\cdot\cos\theta = 3\cos\theta + 2\cos\theta\cos\theta$

$x'(\theta) = -3\sin\theta - 2\sin\theta\cos\theta - 2\cos\theta\sin\theta$

$y = r\sin\theta = (3 + 2\cos\theta)\cdot\sin\theta = 3\sin\theta + 2\cos\theta\sin\theta$

$y'(\theta) = 3\cos\theta - 2\sin\theta\sin\theta + 2\cos\theta\cos\theta$

$$\frac{dy}{dx} = \frac{3\cos\left(\frac{\pi}{2}\right) - 2\sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) + 2\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)}{-3\sin\left(\frac{\pi}{2}\right) - 2\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) - 2\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)} = \frac{0 - 2 + 0}{-3 - 0 - 0} = \boxed{\frac{2}{3}}$$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

find $\frac{d\theta}{dt}$

$$\frac{dr}{dt} = 3 = -2\sin\theta \cdot \frac{d\theta}{dt}$$

$$3 = -2\sin\left(\frac{\pi}{3}\right)\frac{d\theta}{dt}$$

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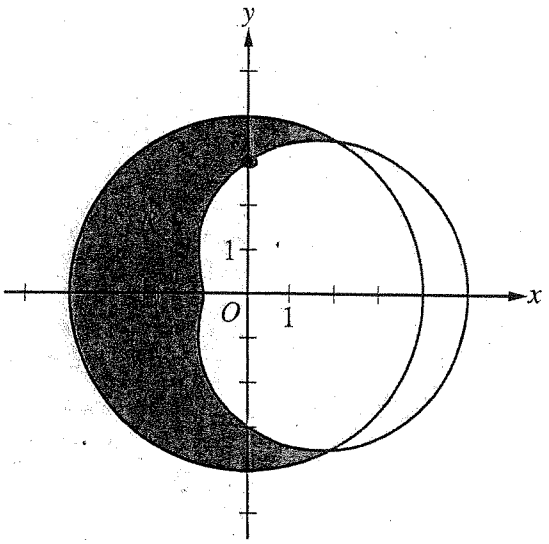

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$R = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \frac{1}{2}\left[4^2 - (3+2\cos\theta)^2\right] d\theta$$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

Slope $= \frac{\Delta y}{\Delta x}$ $\quad r = 3 + 2\cos\theta$

$X = r\cos(\theta) \cdot (3 + 2\cos\theta)$

$Y = \sin(\theta) \cdot (3 + 2\cos\theta)$

$\frac{Y}{X} = \frac{\sin\theta(3+2\cos\theta)}{\cos\theta(3+2\cos\theta)} = \tan\theta$

$\frac{dy}{dx} = 1$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$\frac{dr}{dt} = 3$ units per second

$r = 3 + 2\cos\theta$

$\frac{dr}{dt} = \frac{d(3+2\cos\theta)}{dt}$

$3 \frac{\text{units}}{\text{second}} = 2\frac{d\cos\theta}{dt}$

$3 = 2 \cdot -\sin\theta \cdot \frac{d\theta}{dt}$

$\frac{3}{2} = -\sin\left(\frac{\pi}{3}\right)\frac{d\theta}{dt}$

$\frac{3}{2} = -\frac{\sqrt{3}}{2} \cdot \frac{d\theta}{dt}$

$\frac{-3}{\sqrt{3}} = \frac{d\theta}{dt}$

$-\frac{3\sqrt{3}}{3} = -\sqrt{3}$ radians per second $= \frac{d\theta}{dt}$

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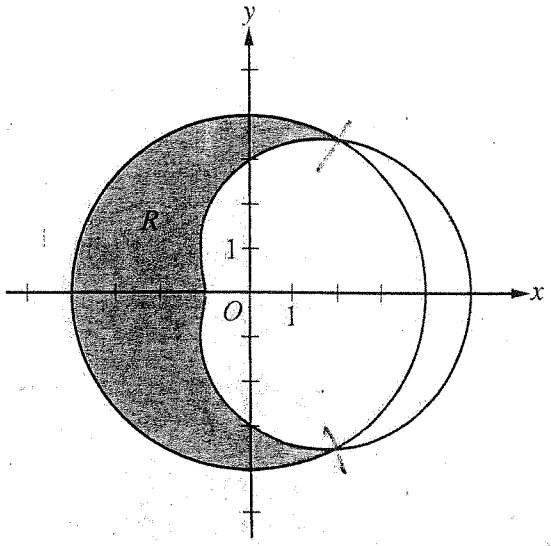

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$3 + 2\cos\theta = 4$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$

$$\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left((4)^2 - (3 + 2\cos\theta)^2 \right) d\theta$$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$r = 3 + 2\cos\theta$

$x = (3 + 2\cos\theta)\cos\theta = 3\cos\theta + 2\cos^2\theta$

$y = (3 + 2\cos\theta)\sin\theta$

$\frac{dx}{d\theta} = (-3\sin\theta - 4\cos\theta\sin\theta)$

$\frac{dy}{d\theta} = (-2\sin\theta)(\sin\theta) + (\cos\theta)(3 + 2\cos\theta)$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{(-2\sin\theta)(\sin\theta) + (\cos\theta)(3 + 2\cos\theta)}{(-3\sin\theta - 4\cos\theta\sin\theta)}$

$\frac{dy}{dx} = \frac{(-2(1))(1) + (0)}{(-3(1) - 0)} = \frac{-2}{-3} = \frac{2}{3}$

$\frac{dy}{dx} = \frac{2}{3}$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$\frac{dr}{dt} = 3$

$r = 3 + 2\cos\theta$

$\frac{dr}{d\theta} = -2\sin\theta$

$3 = -2\sin\theta$

$\frac{3}{-2} = \sin\theta$

$\frac{d\theta}{dt}$ $\frac{d\theta}{dx}$

$x = r\cos\theta$

$r = \frac{x}{\cos\theta}$

$\frac{\cos\theta}{x} = 3 + 2\cos\theta$

$\cos\theta = 3x + 2x\cos\theta$

$x = \frac{\cos\theta}{3 + 2\cos\theta}$

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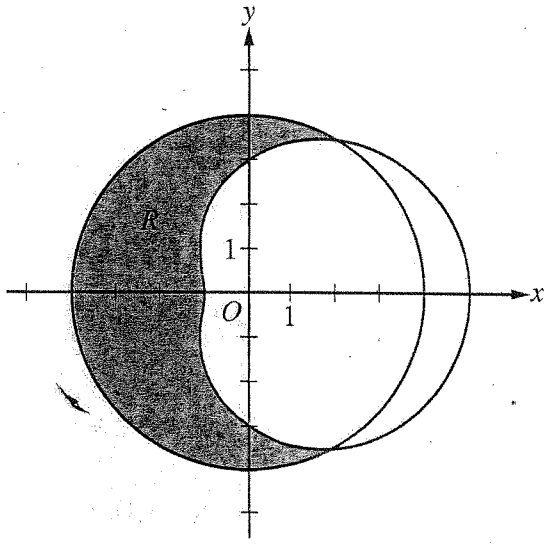

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$R = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} 4^2 - (3+2\cos\theta)^2 \, d\theta$$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$x = r\cos\theta$

$y = r\sin\theta$

$\frac{dx}{d\theta} = -r\sin\theta = -3$

$\frac{dy}{d\theta} = r\cos\theta = 0$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$\frac{dr}{dt} = 3$ $\theta = \frac{\pi}{3}$

$\frac{dr}{dt} = -2\sin\theta \cdot \frac{d\theta}{dt}$

$-2 \cdot \frac{\sqrt{3}}{2}$

$3 = \frac{dr}{dt} = -\sqrt{3} \cdot \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{3}{-\sqrt{3}}$

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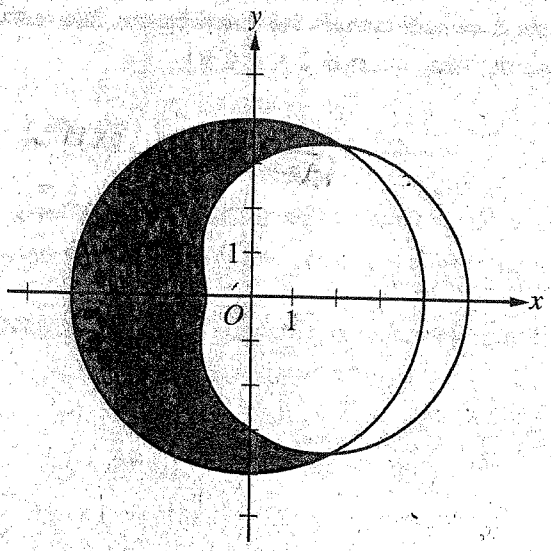

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(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$R = \frac{1}{2}\cdot 2\pi \int_{\pi/3}^{5\pi/3} \left(4^2 - (3 + 2\cos\theta)^2\right) d\theta$$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$\frac{dr}{dt} = -2\sin\theta$

$r'(\pi/2) = -2\cdot\sin\pi/2 = -2$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$\frac{dr}{dt} = -2\sin\theta \cdot \frac{d\theta}{dt}$

$-2\sin\frac{\pi}{3}\frac{d\theta}{dt} = 3$

$\frac{\sqrt{3}}{2} = -\frac{3}{2}\frac{d\theta}{dt}$

$\frac{dr}{dt} = -2\sin\theta$

$-2\sin\theta\frac{d\theta}{dt} =$

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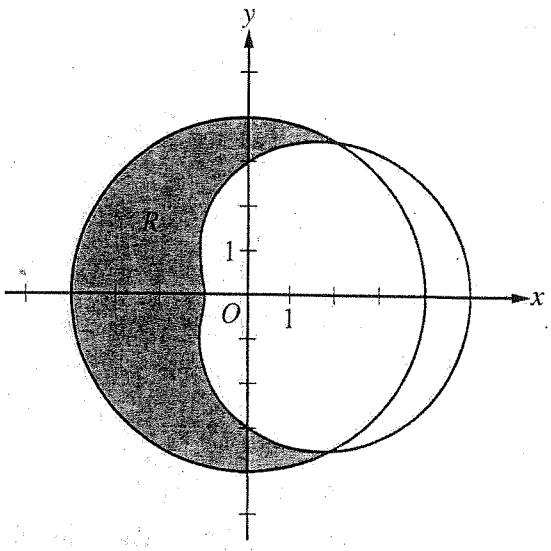

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(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$4 = 3 + 2\cos\theta \quad \frac{1}{2} = \cos\theta \quad \frac{\pi}{3}, \frac{5\pi}{3} = \theta$

$$\int_{-\pi/3}^{\pi/3} 4 - 3 - 2\cos\theta \, d\theta$$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$\frac{dr}{d\theta} = -2\sin\theta$

$\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{2}} = -2 \cdot -1 = \boxed{2}$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

distance btwn particle and origin = r

$r'(t) = 3$

when $\theta = \frac{\pi}{3}$, $r = 3 + 2\cos\frac{\pi}{3} = 3 + 2\left(\frac{1}{2}\right) = 3$

$r = 3 + 2\cos\theta$

$r'(t) = -2\sin\theta \cdot \theta'(t)$

$3 = -2\sin\left(\frac{\pi}{3}\right) \cdot \theta'(t)$

$3 = -2 \cdot \frac{\sqrt{3}}{2} \cdot \theta'(t)$

$-\frac{3}{\sqrt{3}} = \theta'(t)$

$\boxed{-\sqrt{3} = \theta'(t)}$

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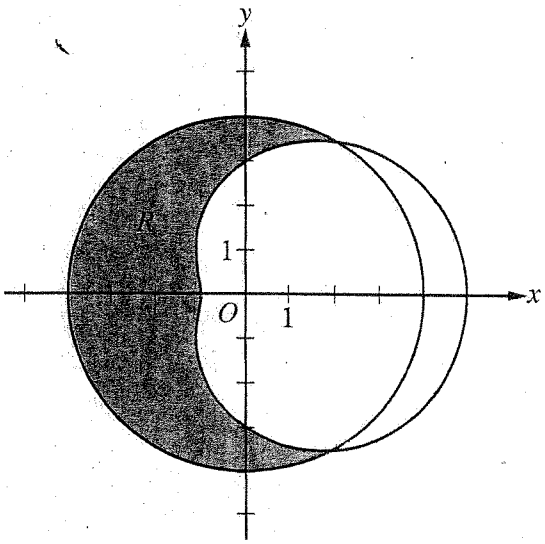

5. The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect at $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

$$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (3+2\cos\theta) - 4$$

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Continue question 5 on page 19.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \frac{\pi}{2}$.

$r = 3 + 2\cos\theta$

$r = -2\sin\theta$

$\boxed{r = -2}$

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \frac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ changes with respect to time at the instant when the position of the particle corresponds to $\theta = \frac{\pi}{3}$. Indicate units of measure.

$$\int_0^{\pi/2} 3 + 2\cos\theta = 3\theta + \cos^2\theta \Big|_0^{\frac{\pi}{2}} = \frac{3\pi}{2} - 1 =$$

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AP® SAMPLE STUDENT RESPONSES AND SCORING NOTES
2018 AP CALCULUS BC

Question 5

Sample Identifier: A
Score: 9

- **The response earned 3-3-3.**
- In part (a) the response earned the first point because the correct constant of $\frac{1}{2}$ and the correct limits of integration of $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ are given. The response earned the second and third points because the correct integrand of $16 - (3 + 2\cos\theta)^2$ is given. The response gives 16 instead of 4^2, but this is not an oversimplification and still earns the second point.
- In part (b) the response earned the first point either because $dx/d\theta$ is computed correctly in the third line on the left side or because $dy/d\theta$ is computed correctly in the third line on the right side. The response earned the second point because dy/dx is assembled correctly in the sixth line. Although dy/dx is not assembled symbolically, $dx/d\theta$ and $dy/d\theta$ are correctly evaluated at $\theta = \frac{\pi}{2}$ in the fourth and fifth lines, and these numeric values are correctly assembled to create dy/dx. The response would have earned the third point with no simplification because the correct answer of $\frac{-2}{-3}$ is given in the sixth line. The response chose to simplify, did so correctly in the sixth and seventh lines, and thus earned the third point.
- In part (c) the response earned the first point because $dr/dt = -2\sin\theta \cdot d\theta/dt$ is presented correctly in the second line using the Chain Rule. The response earned the second point because $d\theta/dt = \frac{-3}{2} \cdot \frac{1}{\sqrt{3}/2}$ is isolated correctly in an equation in the fourth line. $d\theta/dt$ is isolated numerically after $\theta = \frac{\pi}{3}$ is substituted in to the equation in the third line. The response would have earned the third point with no simplification because the correct answer with units of $\frac{-3}{\sqrt{3}}$ radians per second is given in the last line. The response chose to simplify, did so correctly in the last line, and thus earned the third point.

Sample Identifier: B
Score: 8

- **The response earned 3-3-2.**
- In part (a) the response earned the first point because the correct constant of $\frac{1}{2}$ and the correct limits of integration of $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ are given. The response earned the second and third points because the correct integrand of $4^2 - (3 + 2\cos\theta)^2$ is given.
- In part (b) the response earned the first point either because $dx/d\theta$ is computed correctly in the fifth line on the left side or because $dy/d\theta$ is computed correctly in the last line on the left side. The response earned the second point because dy/dx is assembled correctly in the first line on the right side. The response would have earned the third point with no simplification because the correct answer of $\frac{3(0)+2(0)-2(1)}{-3(1)-4(0)(1)}$ is given in the second line on the right side. The response chose to simplify, did so correctly in the third and fourth lines on the right side, and thus earned the third point.
- In part (c) the response earned the first point because $dr/dt = -2\sin\theta \cdot d\theta/dt$ is presented correctly using the Chain Rule in the second line on the left side. The response earned the second point because $d\theta/dt = -3/\sqrt{3}$ is isolated correctly in an equation in the last line on the left side. The response did not earn the third point because the answer in the last line on the right side does not have the correct units of radians per second.

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Question 5 (continued)

Sample Identifier: C
Score: 7

- **The response earned 3-3-1.**
- In part (a) the response earned the first point because the correct constant of $\frac{1}{2}$ (attached to each integrand piece) and the correct limits of integration of $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ are given in the first line. The response earned the second and third points because the correct integrand of $4^2 - (3 + 2\cos\theta)^2$ is given in the first line.
- In part (b) the response earned the first point either because $dx/d\theta = x'(\theta)$ is computed correctly in the second line or because $dy/d\theta = y'(\theta)$ is computed correctly in the fourth line. The response earned the second point because a numerical form of dy/dx is assembled correctly in the last line. The response earned the third point with this same numerical form of dy/dx in the last line.
- In part (c) the response earned the first point because $dr/dt = 3 = -2 \cdot \sin\theta \cdot d\theta/dt$ is presented correctly using the Chain Rule. The response did not earn the second point because $d\theta/dt$ is not isolated in an equation. The response did not earn the third point because the correct answer with units is not given.

Sample Identifier: D
Score: 6

- **The response earned 3-0-3.**
- In part (a) the response earned the first point because the correct constant of $\frac{1}{2}$ and the correct limits of integration of $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ are given. The response earned the second and third points because the correct integrand of $4^2 - (3 + 2\cos\theta)^2$ is given.
- In part (b) the response did not earn the first point because although $x = \cos\theta \cdot (3 + 2\cos\theta)$ and $y = \sin\theta \cdot (3 + 2\cos\theta)$ are correct in the second and third lines on the left side, neither $dx/d\theta$ nor $dy/d\theta$ is computed. The response did not earn the second point because dy/dx is never assembled. The response did not earn the third point because an incorrect answer of 1 is given.
- In part (c) the response earned the first point because $dr/dt = 3 = 2 \cdot -\sin\theta \cdot d\theta/dt$ is presented correctly using the Chain Rule in the first line on the right side. The response earned the second point because $d\theta/dt$ is isolated correctly in the equation in the next to last line on the right side. The response earned the third point because the correct answer with units is given in the last line on the right side.

Sample Identifier: E
Score: 5

- **The response earned 2-3-0.**
- In part (a) the response did not earn the first point because the limits of integration are incorrect. The response earned the second and third points because the correct integrand of $4^2 - (3 + 2\cos\theta)^2$ is given in the last line.
- In part (b) the response earned the first point either because $dx/d\theta$ is computed correctly in the fourth line on the left side or because $dy/d\theta$ is computed correctly in the fifth line on the left side. The response earned the second point because dy/dx is assembled correctly in the expression $\frac{dy/d\theta}{dx/d\theta}$ in the fourth line on the right side. The response would earned the third point with no

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Question 5 (continued)

simplification because the correct answer of $\frac{(-2(1))(1)+0}{(-3(1)-0)}$ is given in the last line on the right side. The response chose to simplify, did so correctly in the last line on the right side, and thus earned the third point.

- In part (c) the response did not earn any points because the response does not present an application of the Chain Rule to $r = 3 + 2\cos\theta$ and therefore is not eligible for any points.

Sample Identifier: F
Score: 4

- **The response earned 2-0-2.**
- In part (a) the response did not earn the first point because an incorrect constant of π is given. The response earned the second and third points because the correct integrand of $4^2 - (3 + 2\cos\theta)^2$ is given.
- In part (b) the response did not earn the first point because although $x = r\cos\theta$ is correctly defined in the first line and $y = r\sin\theta$ is correctly defined in the second line, neither $dx/d\theta$ nor $dy/d\theta$ is computed correctly. The response did not earn the second point because dy/dx is never assembled. The response did not present an answer and therefore did not earn the third point.
- In part (c) the response earned the first point because $dr/dt = -2\sin\theta \cdot d\theta/dt$ is presented correctly using the Chain Rule in the second line. The response earned the second point because $d\theta/dt$ is isolated correctly in the equation on the right side. The response did not earn the third point because the answer circled on the right side does not present the units of radians per second.

Sample Identifier: G
Score: 3

- **The response earned 2-0-1.**
- In part (a) the response did not earn the first point because an incorrect constant is given even in the presence of the correct limits of integration. The response earned the second and third points because the correct integrand of $4^2 - (3 + 2\cos\theta)^2$ is given.
- In part (b) the response did not earn any points. The response did not earn the first point because neither $dx/d\theta$ nor $dy/d\theta$ is computed. Therefore, the response is not eligible for the second and third points.
- In part (c) the response earned the first point because $dr/dt = -2\sin\theta \cdot d\theta/dt$ is presented correctly using the Chain Rule in the first line. The response did not earn the second point because $d\theta/dt$ is not isolated in an equation. The response did not earn the third point because the correct answer with units is not given.

Sample Identifier: H
Score: 2

- **The response earned 0-0-2.**
- In part (a) the response did not earn the first point because incorrect limits of integration are given in the second line. The response did not earn the second and third points because the integrand in the second line is incorrect.
- In part (b) the response did not earn any points. The response did not earn the first point because neither $dx/d\theta$ nor $dy/d\theta$ is computed. Therefore, the response is not eligible for the second and third points.

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Question 5 (continued)

- In part (c) the response earned the first point because $r'(t) = -2\sin\theta \cdot \theta'(t)$ is presented correctly using the Chain Rule in the fifth line. The response earned the second point because $\theta'(t)$ is isolated correctly in the equation in the eighth line. The response did not earn the third point because units are never presented with the correct answer.

Sample Identifier: I
Score: 1

- **The response earned 1-0-0.**
- In part (a) the response earned the first point because the correct constant of $\frac{1}{2}$ and the correct limits of integration of $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ are given. The response did not earn the second and third points because the integrand is incorrect.
- In part (b) the response did not earn any points. The response did not earn the first point because neither $dx/d\theta$ nor $dy/d\theta$ is computed. Therefore, the response is not eligible for the second and third points.
- In part (c) the response did not earn any points because the response does not present a derivative of $r = 3 + 2\cos\theta$ and therefore is not eligible for any points.

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6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $\left|P_4(2) - f(2)\right|$.

STOP

END OF EXAM

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Question 6

(a) The first four nonzero terms are $\frac{x^2}{3} - \frac{x^3}{2 \cdot 3^2} + \frac{x^4}{3 \cdot 3^3} - \frac{x^5}{4 \cdot 3^4}$.

The general term is $(-1)^{n+1} \frac{x^{n+1}}{n \cdot 3^n}$.

2 : { 1 : first four terms; 1 : general term }

(b) $\lim_{n \to \infty} \left| \frac{\frac{(-1)^{n+2} x^{n+2}}{(n+1)\left(3^{n+1}\right)}}{\frac{(-1)^{n+1} x^{n+1}}{n \cdot 3^n}} \right| = \lim_{n \to \infty} \left| \frac{-x}{3} \cdot \frac{n}{(n+1)} \right| = \left| \frac{x}{3} \right|$

$\left| \frac{x}{3} \right| < 1$ for $|x| < 3$

Therefore, the radius of convergence of the Maclaurin series for f is 3.

5 : { 1 : sets up ratio; 1 : computes limit of ratio; 1 : radius of convergence; 1 : considers both endpoints; 1 : analysis and interval of convergence }

— OR —

The radius of convergence of the Maclaurin series for $\ln(1+x)$ is 1, so the series for $f(x) = x \ln\left(1 + \frac{x}{3}\right)$ converges absolutely for $\left| \frac{x}{3} \right| < 1$.

$\left| \frac{x}{3} \right| < 1 \Rightarrow |x| < 3$

Therefore, the radius of convergence of the Maclaurin series for f is 3.

— OR —

5 : { 1 : radius for $\ln(1+x)$ series; 1 : substitutes $\frac{x}{3}$; 1 : radius of convergence; 1 : considers both endpoints; 1 : analysis and interval of convergence }

When $x = -3$, the series is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-3)^{n+1}}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{3}{n}$, which diverges by comparison to the harmonic series.

When $x = 3$, the series is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^{n+1}}{n \cdot 3^n} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n}$, which converges by the alternating series test.

The interval of convergence of the Maclaurin series for f is $-3 < x \leq 3$.

(c) By the alternating series error bound, an upper bound for $|P_4(2) - f(2)|$ is the magnitude of the next term of the alternating series.

$|P_4(2) - f(2)| < \left| -\frac{2^5}{4 \cdot 3^4} \right| = \frac{8}{81}$

2 : { 1 : uses fifth-degree term as error bound; 1 : answer }

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A

6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$$\ln\left(1+\frac{x}{3}\right) = \frac{x}{3} - \frac{x^2}{2\cdot 3^2} + \frac{x^3}{3\cdot 3^3} - \frac{x^4}{4\cdot 3^4} + \cdots$$

$$\downarrow$$

$$f(x) = x\ln\left(1+\frac{x}{3}\right) = \frac{x^2}{3} - \frac{x^3}{2\cdot 3^2} + \frac{x^4}{3\cdot 3^3} - \frac{x^5}{4\cdot 3^4} + \cdots + \frac{(-1)^{n+1}x^{n+1}}{n\cdot 3^n} + \cdots$$

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Continue question 6 on page 21.

A

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+1}}{n\, 3^n}$$

ratio test: $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\frac{x^{n+2}}{(n+1)3^{n+1}}}{\frac{x^{n+1}}{n3^n}}\right| = \lim_{n\to\infty} \left|\frac{x(n)}{(n+1)3}\right|$

$= \left|\frac{x}{3}\right| < 1 \rightarrow$ radius: $-3 < x < 3$

if $x = -3 \rightarrow \sum \frac{3}{n}$ diverges by harmonic series

if $x = 3 \rightarrow \sum \frac{3(-1)^{n+1}}{n}$ converges by alternating series test.

interval of convergence:

$$\boxed{-3 < x \leq 3}$$

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$|R_4(2)| = |P_4(2) - f(2)|$

$$\frac{2^5}{4 \cdot 3^4} = \frac{2^3}{3^4} = \frac{8}{81}$$

$|R_4(2)| \leq \frac{8}{81}$

$|P_4(2) - f(2)| \leq \boxed{\frac{8}{81}}$

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6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$x^3/27$ $x^4/81$

$$x\left(\frac{x}{3} - \frac{(x/3)^2}{2} + \frac{(x/3)^3}{3} - \frac{(x/3)^4}{4} + \cdots\right)$$

$$\downarrow$$

$$\frac{x^2}{3} - \frac{x^3}{3^2\cdot 2} + \frac{x^4}{3^3\cdot 3} - \frac{x^5}{3^4\cdot 4} + \cdots + (-1)^{n+1}\frac{x^{n+1}}{3^n n} + \cdots$$

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Continue question 6 on page 21.

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+2}x^{n+2}}{3^{n+1}(n+1)}\cdot\frac{3^n n}{(-1)^{n+1}x^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{nx}{3(n+1)}\right| = \left|\frac{x}{3}\right| < 1$$

$(-1)^{n+1}\quad (-1)^{n+1}\quad (3)^{n+1}$

$-1 < \frac{x}{3} < 1$

$-3 < x < 3$

$x=-3\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(-3)^{n+1}}{3^n n} = \sum_{n=1}^{\infty}\frac{(-1)^{2n+2}(3)^{n+1}}{3^n n} \to \sum_{n=1}^{\infty}\frac{3^{n+1}}{n(\frac{1}{3})(3^{n+1})} \to \sum_{n=1}^{\infty}\frac{3}{n}\left(\frac{3}{3}\right)^{n+1}$

let $\sum_{n=1}^{\infty}\frac{3}{n}(1)^{n+1} = a_n$. a_n is positive and decreasing and $\lim_{n\to\infty} a_n = 0$, thus it converges by the Alternating Series test

$x=3\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(3)^{n+1}}{3^n n} \to \sum_{n=1}^{\infty}\frac{3^{n+1}}{n(\frac{1}{3})(3)^{n+1}} \to \sum_{n=1}^{\infty}\frac{3}{n}\left(\frac{3}{3}\right)^{n+1}$

this series converges for the same reasons as the one above

Thus, f converges for $-3 \le x \le 3$

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$$\frac{2^5}{3^4 \cdot 4} = \frac{32}{324}$$

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C

6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\ln\left(1+\frac{x}{3}\right) = \frac{x}{3} - \frac{\left(\frac{x}{3}\right)^2}{2} + \frac{\left(\frac{x}{3}\right)^3}{3} - \frac{\left(\frac{x}{3}\right)^4}{4} + \cdots$$

$$= \frac{x}{3} - \frac{x^2}{3^2\cdot 2} + \frac{x^3}{3^3\cdot 3} - \frac{x^4}{3^4\cdot 4} + \cdots$$

$$x\ln\left(1+\frac{x}{3}\right) = \frac{x^2}{3} - \frac{x^3}{3^2\cdot 2} + \frac{x^4}{3^3\cdot 3} - \frac{x^5}{3^4\cdot 4} + \cdots$$

1 2 3 4

$$+ \cdots + (-1)^{n+1}\frac{x^{n+1}}{3^n\cdot n} + \cdots$$

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Continue question 6 on page 21.

C

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+1}}{3^n \cdot n}$ Ratio Test

$\lim_{n\to\infty} \frac{|x^{n+2}|}{3^{n+1}\cdot(n+1)} \cdot \frac{3^n \cdot n}{|x^{n+1}|}$

$\lim_{n\to\infty} \frac{|x^n||x^2| 3^n \cdot n}{3^n \cdot 3 \cdot (n+1)|x^n||x|} = \frac{|x|}{3}$

$\frac{|x|}{3} < 1$

$|x| < 3$

$-3 < x < 3$

$\boxed{-3 < x \leq 3}$

when $x = 3$

$\frac{(-1)^{n+1} 3^{n+1}}{3^n \cdot n}$

$\frac{3^2}{3 \cdot 1} = \frac{3}{1}$ $\quad \frac{-1 \cdot 3^3}{3^2 \cdot 2} = \frac{-3}{2}$ $\quad \frac{3}{3}$ $\quad \frac{-3^5}{3^4 \cdot 4} = -3/4$

$x = 3$ converges by the AST (terms decrease, signs alternate $\lim_{n\to\infty} = 0$)

when $x = -3$

$\frac{(-1)^{n+1} (-3)^{n+1}}{3^n \cdot n}$

$\frac{(-3)^2}{3 \cdot 1} = 3$ $\quad \frac{-(-3)^3}{3^2 \cdot 2} = \frac{3}{2}$ $\quad \frac{(-3)^4}{3^3 \cdot 3} = \frac{3}{3}$ $\quad 3/4$ $\quad \frac{(-3)^6}{3^5 \cdot 5} = \frac{3}{5}$

$x = -3$ diverges because the partial sums do not approach a limit and by the n^{th} term test

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$|P_4(2) - f(2)| \leq P_5(2)$

$\frac{-x^5}{3^4 \cdot 4} = \frac{-2^5}{3^4 \cdot 4}$

$|P_4(2) - f(2)| \leq \frac{-32}{3^4 \cdot 4}$

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D

6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$$P_4(x) = x\left(x - \frac{\left(\frac{x}{3}\right)^2}{2} + \frac{\left(\frac{x}{3}\right)^3}{3} - \frac{\left(\frac{x}{3}\right)^4}{4}\right)$$

$$= \frac{x^2}{3} - \frac{x^3}{2\cdot 3^2} + \frac{x^4}{3\cdot 3^3} - \frac{x^5}{4\cdot 3^4}$$

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Continue question 6 on page 21.

D

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

Maclaurin series of $f = \sum_{n=1}^{\infty} \frac{x^{n+1}(-1)^{n+1}}{(n)(3^n)}$

$$\lim_{n\to\infty} \left| \frac{x^{n+2}}{(n+1)(3^{n+1})} \cdot \frac{n(3^n)}{(x^{n+1})} \right| = \lim_{n\to\infty} \left| \frac{x(n)}{3(n+1)} \right| = \left|\frac{x}{3}\right| = L < 1$$

$-1 < \frac{x}{3} < 1$

$-3 < x < 3$ $\quad 2n+2$

Endpts:

(a) $x=-3$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}(3)^{n+1}}{n(3^n)} = \sum_{n=1}^{\infty} \frac{3}{n}$ diverges by p-series test $(p=1 \leq 1)$

(a) $x=3$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3)^{n+1}}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}3}{n}$

① $\lim_{n\to\infty} \frac{3}{n} = 0$

② $\frac{3}{n+1} \overset{?}{<} \frac{3}{n}$

$\therefore$ Series is convergent at $x=3$

by alternating series test.

interval of convergence : $(-3, 3]$

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$. $f(2) = 2\ln(5/3)$

$P_4(2) = \frac{4}{3} - \frac{4}{9} + \frac{16}{3^4} - \frac{32}{4 \cdot 3^4}$

$= \frac{4}{3} - \frac{4}{9} + \frac{16}{81} - \frac{32}{324}$

Upper bound $= \left| \left(\frac{4}{3} - \frac{4}{9} + \frac{16}{81} - \frac{32}{324}\right) - 2\ln(5/3) \right| = \left| \frac{352}{324} - (2\ln(5/3)) \right|$

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E

6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$\frac{x^2}{3} - \frac{x^3}{6} + \frac{x^4}{9} - \frac{x^5}{12} + \cdots (-1)^{n+1}\frac{x^{n+1}}{3n}$

$\sum_{n=1}^{\infty} (-1)^{n+1}\frac{x^{n+1}}{3n}$

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Continue question 6 on page 21.

E

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

Ratio Test $\left| \frac{(-1)^{n+2} x^{n+2}}{3n+3} \cdot \frac{3n}{x^{n+1}(-1)^{n+1}} \right|$

$\left| \frac{-3nx}{3n+3} \right| < 1$

$|-x| < 1$

$-1 < x \leq 1$

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(1)^{n+1}}{3n}$

$\lim_{n \to \infty} \left| \frac{(-1)^{n+1}}{3n} \right| = 0$

converges by alternating series test

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}}{3n}$

$\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{1}{n}\right)$

diverges by p-series test

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$P_5(x) \leq -\frac{x^5}{12}$

$P_5(2) \leq -\frac{2^5}{12} = -\frac{32}{12} = -\frac{8}{3}$

$|P_4(2) - f(2)| < \frac{8}{3}$

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6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$\frac{x^2}{9}\cdot\frac{1}{2} = \frac{x^2}{18}$

$\frac{x^3}{27}$

$\frac{x^4}{81}$

$\frac{x^4}{324}$

$p(x) = x\left(\frac{x}{3} - \frac{\left(\frac{x}{3}\right)^2}{2} + \frac{\left(\frac{x}{3}\right)^3}{3} - \frac{\left(\frac{x}{3}\right)^4}{4}\right) + \cdots$

$p(x) = x\left(\frac{x}{3} - \frac{x^2}{18} + \frac{x^3}{81} - \frac{x^4}{324}\right) + \cdots$

$p(x) = \frac{x^2}{3} - \frac{x^3}{18} + \frac{x^4}{81} - \frac{x^5}{324} + \cdots + \frac{(-1)^{n+1}\left(\frac{x}{3}\right)^{n+1}}{n} + \cdots$

$n=1$ $n=2$

$(-1)^{n+1}\cdot\frac{\left(\frac{x}{3}\right)^{n+1}}{n}$

$\frac{(-1)^{n+1}\,x^{n+1}}{3\cdot n!}$

$n=3 \quad \frac{x^4}{\left(\frac{x}{3}\right)^3 \cdot n}$

$\frac{\left(\frac{x}{3}\right)^{n+1}}{n} \quad \frac{\left(\frac{x}{3}\right)^3}{3}$

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Continue question 6 on page 21.

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

Ratio Test:

$$\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+2}\left(\frac{x}{3}\right)^{n+2}}{n+1}}{\frac{(-1)^{n+1}\left(\frac{x}{3}\right)^{n+1}}{n}}\right| = \lim_{n\to\infty}\left|\frac{(n)(-1)^{n+2}\left(\frac{x}{3}\right)^{n+2}}{(n+1)(-1)^{n+1}\left(\frac{x}{3}\right)^{n+1}}\right| =$$

$$\lim_{n\to\infty}\left|\frac{n(-1)\left(\frac{x}{3}\right)}{(n+1)}\right|$$

$$\left|-\frac{x}{3}\right| < 1$$

$$-1 < \frac{x}{3} < 1 \cdot 3$$

$$\boxed{-3 < x < 3}$$

Checking endpoints:

$x=-3$: $\sum \frac{n}{n+1}$

$\lim_{n\to\infty}\frac{n}{n+1} = 1$ diverges, $1 \neq 0$

$x=3$: $\sum \frac{-n}{n+1}$

$\lim_{n\to\infty}\frac{-n}{n+2} = -1$ diverges, $-1 \neq 0$

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$. $n=5$

$$|P_4(2) - f(2)| < \frac{(-1)^6\left(\frac{2}{3}\right)^6}{5} = \frac{64}{3645}$$

$$\frac{2}{3}\cdot\frac{2}{3} = \frac{4}{9}\cdot\frac{2}{3} = \frac{8}{27}\cdot\frac{2}{3} = \frac{16}{81}\cdot\frac{2}{3} = \frac{32}{243}\cdot\frac{2}{3} = \frac{64}{729}\cdot\frac{1}{5} = \frac{64}{3645}$$

81
1 x3/243
2
1 x/3
2 x2/9
x5/3645

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6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$$\ln\left(1+\frac{x}{3}\right) = \frac{x}{3} - \frac{x^2}{6} + \frac{x^3}{9} - \frac{x^4}{12} + \cdots + \frac{(-1)^{n+1}x^n}{3n} + \cdots$$

$$f(x) = x\ln\left(1+\frac{x}{3}\right) = \frac{x^2}{3} - \frac{x^3}{6} + \frac{x^4}{9} - \frac{x^5}{12} + \cdots + \frac{(-1)^{n+1}x^{n+1}}{3n} + \cdots$$

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Continue question 6 on page 21.

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{n+1}}{3n}$

$\lim_{n\to\infty} \left| \frac{x^{n+2}}{3(n+1)} \cdot \frac{3n}{x^{n+1}} \right|$

$= \lim_{n\to\infty} \left| \frac{3n}{3(n+1)} \cdot x \right| = |x| < 1$

$-1 < x < 1$

$x = -1 \quad \sum_{n=1}^{\infty} \frac{1}{3n}$ harmonic series - diverges

$x = 1 \quad \sum_{n=1}^{\infty} \frac{x^2}{3n}$

$x = -1 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}}{3n}$

AST $\sum \frac{1}{3n}$

$\lim_{n\to\infty} \frac{1}{3n} = 0$ ✓ converge by AST

$\frac{1}{3(n+1)} \leq \frac{1}{3n}$ ✓

$x = 1 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n}$

converges by AST

$-1 \leq x \leq 1$

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

1st omitted term $= -\frac{x^5}{12}$

$= -\frac{1}{12}$

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6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$f(x) = x\ln\left(1+\frac{x}{3}\right)$

$\ln(1+x) = (-1)^{n+1}\frac{x^n}{n}$

general term $\frac{(-1)^{n+1}(x)^{2n}}{3n}$

four nonzero terms: $\frac{x^2}{3} - \frac{x^3}{6} + \frac{x^4}{9} - \frac{x^5}{12}$

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Continue question 6 on page 21.

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+2}(x)^{2n+2}}{3n+3}\cdot\frac{3n+}{(-1)^{2n+1}(x)^{2n}}\right|$$

$$\lim_{n\to\infty}\left|\frac{x(3n)}{3n+3}\right| \quad \cdot \quad x<1 \qquad R=1 \quad C=0$$

$-1 \quad 0 \quad 1$

$x=-1$ $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(-1)^{2n}}{3n} = \sum_{n=0}^{\infty}\frac{(-1)^{3n+1}}{3n}$ → converges due to Alt. Series Test

$x=1$ $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(1)^{2n}}{3n} = \sum_{n=0}^{\infty}\frac{(-1)^{3n+1}}{3n}$ → converges due to Alt Series Test

IOC: $[-1, 1]$

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$$\left|\frac{2^5}{12} - 2\ln\left(1+\frac{2}{3}\right)\right|$$

$$32 - 2\ln\left(\frac{5}{3}\right)$$

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6. The Maclaurin series for $\ln(1+x)$ is given by

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots.$$

On its interval of convergence, this series converges to $\ln(1+x)$. Let f be the function defined by $f(x) = x\ln\left(1+\frac{x}{3}\right)$.

(a) Write the first four nonzero terms and the general term of the Maclaurin series for f.

$x \ln\left(1+\frac{x}{3}\right) = x\left(\frac{x}{3} - \frac{x^2}{6} + \frac{x^3}{9} - \frac{x^4}{12} + \cdots\right)$ ($\ln\left(1+\frac{x}{3}\right)$)

$\therefore \quad \frac{x^2}{3} - \frac{x^3}{6} + \frac{x^4}{9} - \frac{x^5}{12} + \cdots + (-1)^{n+1}\frac{x^{n+1}}{3n} + \cdots$

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Continue question 6 on page 21.

(b) Determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

$$\left| \frac{x^{n+1}}{3(n+1)} \cdot \frac{3(n)}{x^{n+1}} \right| = \lim_{n\to\infty} \left| \frac{3nx}{3n+3} \right| = \left| \frac{\frac{3nx}{n}}{\frac{3n}{n}+\frac{3}{n}} \right| < 1 \quad \left| \frac{3x}{3} \right| < 1$$

$-1 < \frac{3x}{3} < 1 \quad = \quad -1 < x < 1$

Test endpoints

test $\boxed{-1}$

plug $(-1)^{-1+1} \left| \frac{x^{-1+1}}{3(-1)} \right| = \left| \left(\frac{1}{-3}\right) \right| \rightarrow$ convergent

test $\boxed{1}$

plug $(-1)^{1+1} \left(\frac{x^{1+1}}{3(1)} \right) = \frac{x^2}{3} \rightarrow$ divergent

∴ interval of convergence $[-1, 1)$

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Use the alternating series error bound to find an upper bound for $|P_4(2) - f(2)|$.

$f(2) = 2\ln\left(1+\frac{2}{3}\right)$ $\quad P_5(2) = \frac{x^6}{15}\Big|_2 = \frac{2^6}{15} = \frac{64}{15}$

$2\ln\left(1+\frac{2}{3}\right) - \frac{64}{15} < x < 2\ln\left(1+\frac{2}{3}\right) + \frac{64}{15}$

∴ upper bound of error $= 2\ln\left(1\frac{2}{3}\right) + \frac{64}{15}$

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AP® SAMPLE STUDENT RESPONSES AND SCORING NOTES
2018 AP CALCULUS BC

Question 6

Sample Identifier: A
Score: 9

- **The response earned 2-5-2.**
- In part (a) the response earned the first point in the first line as the response correctly gives the first four nonzero terms. The response earned the second point in the second line as the correct general term of the Maclaurin series for f is given.
- In part (b) the response earned the first point in the second line by giving a correct ratio. The response earned the second point in the third line with the correct limit $|x/3|$. The response earned the third point in the third line with the inequality $-3 < x < 3$. The response earned the fourth point in the fourth and fifth lines by substituting $x = 3$ and $x = -3$ into the Maclaurin series for f resulting in the series $\sum \frac{3}{n}$ and $\sum \frac{3(-1)^{n+1}}{n}$. The response earned the fifth point in the fourth, fifth and seventh lines by stating that three times the harmonic series diverges, stating that the series in the fifth line converges by alternating series test, and in the seventh line by giving the correct interval of convergence.
- In part (c) the response earned the first point in the second line with the correct fifth degree term evaluated at 2, $\frac{2^5}{4 \cdot 3^4}$. The response earned the second point in the second line with the correct fifth degree term evaluated at 2, $\frac{2^5}{4 \cdot 3^4}$. Further simplification was correct.

Sample Identifier: B
Score: 8

- **The response earned 2-4-2.**
- In part (a) the response earned the first point in the second line by giving the correct first four nonzero terms of the Maclaurin series for f. The response earned the second point in the second line by giving the correct general term of the Maclaurin series for f.
- In part (b) the response earned the first point in the first line by giving a correct ratio. The response earned the second point in the first line by correctly evaluating the limit as $\left|\frac{x}{3}\right|$. The response earned the third point with the inequality $-3 < x < 3$ on the right. The response earned the fourth point by substituting $x = -3$ and $x = 3$ into the Maclaurin series for f resulting in series $\sum \frac{(-1)^{n+1}(-3)^{n+1}}{3^n \cdot n}$ and $\sum \frac{(-1)^{n+1}(3)^{n+1}}{3^n \cdot n}$. The response did not earn the fifth point as the response stated incorrectly that the series resulting in substituting in $x = -3$ "converges by the Alternating Series test".
- In part (c) the response earned both points by using the fifth degree term $\frac{2^5}{3^4 \cdot 4}$. It is not necessary to simplify but the response did so correctly.

Sample Identifier: C
Score: 7

- **The response earned 2-4-1.**
- In part (a) the response earned the first point in the fourth line by giving the correct first four nonzero terms of the Maclaurin series for f. The response earned the second point in the fifth line by giving the correct general term of the Maclaurin series for f.
- In part (b) the response earned the first point in the second line by giving a correct ratio. The response chose to simplify, did so correctly, and thus earned the second point. The response

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Question 6 (continued)

earned the second point in the third line with the correct limit $\frac{|x|}{3}$. The response earned the third point in the third line on the right with the inequality $-3 < x < 3$. The response earned the fourth point in the lower left by substituting $x = 3$ and $x = -3$ into the general term for f resulting in the terms $\frac{(-1)^{n+1}3^{n+1}}{3^n \cdot n}$ and $\frac{(-1)^{n+1}(-3)^{n+1}}{3^n \cdot n}$. The response did not earn the fifth point as the response provides incorrect justification for divergence at $x = -3$ by saying that the series diverged by the nth term test.

- In part (c) the response earned the first point on the second line by using the fifth degree term $\frac{-x^5}{3^4 \cdot 4}$. The response did not earn the second point as the response presents a negative answer for the upper bound.

Sample Identifier: D
Score: 6

- **The response earned 1-5-0.**
- In part (a) the response earned the first point in the second line by giving a correct first four nonzero terms of the Maclaurin series for f. The response did not earn the second point as the response does not give a general term of the Maclaurin series for f in part (a).
- In part (b) the response earned the first point in the second line by giving a correct ratio. The response earned the second point in the second line with the correct limit $\left|\frac{x}{3}\right|$. The response earned the third point in the fourth line with the inequality $-3 < x < 3$. The response earned the fourth point in the fifth and sixth lines by substituting $x = -3$ and $x = 3$ into the Maclaurin series for f resulting in the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}3^{n+1}}{n(3^n)}$ and $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}3^{n+1}}{n(3^n)}$. The response earned the fifth point in the fifth, seventh and eighth lines by stating that the series when $x = -3$ diverges by p-series test, stating that the series when $x = 3$ in the sixth line converges by alternating series test, and in the eighth line giving the correct interval of convergence.
- In part (c) the response did not earn the first point as the response is using too many terms of the Maclaurin series for f. The response did not earn the second point as they did not give a consistent answer.

Sample Identifier: E
Score: 5

- **The response earned 0-3-2.**
- In part (a) the response did not earn the first point as three of the four terms presented are incorrect for the Maclaurin series for f. The response did not earn the second point as the general term given is incorrect.
- In part (b) the response earned the first point in the first line by giving a correct ratio for the general term presented by the response in part (a). The incorrect general term imported from (a) results in an over-simplification of the problem that renders the response ineligible for the second and third points. The response earned the fourth point in the fourth line by substituting $x = 1$ and $x = -1$ into the presented Maclaurin series for f resulting in the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(1)^{n+1}}{3n}$ and $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}}{3n}$. The response earned the fifth point by stating that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(1)^{n+1}}{3n}$ converges by alternating series test and the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}}{3n}$ diverges by p-series test, and giving the correct interval of convergence for the presented Maclaurin series.

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Question 6 (continued)

- In part (c) the response earned the first point in the first line by presenting the fifth degree term presented in part (a). The response earned the second point in the third line with the consistent value $\frac{8}{3}$.

Sample Identifier: F
Score: 4

- **The response earned 1-3-0.**
- In part (a) the response earned the first point in the third line by giving the correct first four nonzero terms of the Maclaurin series for f. The response did not earn the second point as the response does not give a correct general term of the Maclaurin series for f as the exponent on 3 is incorrect. The error in the general term does not over simplify our problem making the response eligible for points in part (b).
- In part (b) the response earned the first point in the second line by giving a consistent ratio with the general term presented by the response in part (a). The response earned the second point on the fourth line with the correct limit for the ratio presented on the second line. The response earned the third point with the inequality $-3 < x < 3$. The response did not earn the fourth point as $x = 3$ and $x = -3$ are not substituted into the general term presented in (a) for f. The response did not earn the fifth point as an incorrect interval of convergence is given.
- In part (c) the response did not earn the first point as the response uses the sixth degree term and not the fifth degree term. As a result, the response was ineligible for the second point.

Sample Identifier: G
Score: 3

- **The response earned 0-2-1.**
- In part (a) the response did not earn the first point as three of the four terms given are incorrect for the Maclaurin series of f. The response does not give a correct general term for the series and thus did not earn the second point.
- In part (b) the response earned the first point in the second line as the response gives a ratio that is consistent with the general term presented in part (a). The response did not earn the second point as the presented ratio has over-simplified the problem and therefore the response is ineligible for the second point. The response did not earn the third point as the presented limit has over-simplified the problem and therefore the response is ineligible for the third point. The response earned the fourth point as the response substitutes $x = 1$ and $x = -1$ into their general term from part (a) and presents the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n}$ and $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}}{3n}$. The response did not earn the fifth point as the response does not give the correct interval of convergence for their series and the analysis for $x = -1$ is incorrect. Note that the work in the upper right hand corner is crossed out and therefore will not be scored.
- In part (c) the response earned the first point by identifying the fifth degree term $\frac{-x^5}{12}$ consistent with their response in part (a). The response did not earn the second point as the response is incorrect.

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Question 6 (continued)

Sample Identifier: H
Score: 2

- **The response earned 0-2-0.**
- In part (a) the response did not earn the first point as three of the four terms given are incorrect for the Maclaurin series of f. The response does not give a correct general term for the series and thus did not earn the second point.
- In part (b) the response earned the first point in the first line as the response gives a ratio that is consistent with the general term presented in part (a). The incorrect general term imported from (a) results in an over-simplification of the problem that renders the response ineligible for the second and third points. The response earned the fourth point as the response substitutes x=1 and x=-1 into their general term and presents the series $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(-1)^{2n}}{3n}$ and $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(1)^{2n}}{3n}$. The response is not eligible for the fifth point as their series converges at both endpoints which over simplifies the question from the correct solution.
- In part (c) the response did not earn either of the two points as the response is not accurately identifying the error bound.

Sample Identifier: I
Score: 1

- **The response earned 0-1-0.**
- In part (a) the response did not earn the first point as three of the four terms given are incorrect for the Maclaurin series of f. The response did not give a correct general term for the series and thus does not earn the second point.
- In part (b) the response earned the first point in the first line as the response gives a ratio that is consistent with the general term presented in part (a). The incorrect general term imported from (a) results in an over-simplification of the problem that renders the response ineligible for the second and third points. The response did not earn the fourth point as the response incorrectly considers the endpoints by substituting in 1 and -1 in for n where the response should substitute 1 and -1 in for x. The response did not earn the fifth point as the response does not give an interval of convergence consistent with their presented general term from part (a).
- In part (c) the response did not earn the first point as the response uses the sixth degree term from their presented series in part (a) whereas the response should use their fifth degree term. The response did not earn the second point as the response does not give a correct answer consistent with their presented series from part (a).

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